Inflation, Inequality, and the Business Cycle

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Preliminary. Comments welcome.

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Abstract

In this paper we introduce a nonlinear Phillips curve with a state-dependent slope into an otherwise standard HANK model. We show that the nonlinear Phillips curve is crucial for the model to be able to account *jointly* for the properties of inflation and inequality observed in the data. Our model implies that over the business cycle inflation and income inequality increase more strongly than they decrease. Thus, our model is able to account well for the observed positive skewed distribution of US inflation rates and income inequality. A version of our model with a constant Phillips curve slope fails to account for the observed skewness in inflation and inequality.

JEL Classification: E31, E32, E52

Keywords: HANK, inequality, nonlinear Phillips Curve, inflation, skewness

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1 Introduction

The recent surge in inflation rates, following the Covid-19 crisis, caught many economists and central banks by surprise. The inflation rates in the U.S. and other advanced economies were the highest observed in many decades. A similar surprise occurred in the years following the Great Financial Crisis and Great Recession. Then economists and central banks were surprised that inflation rates fell less than predicted at the time. Recent research has shown that standard linearized New Keynesian models have difficulties to explain inflation developments during deep crises, mirroring the surprises by economists and central bank mentioned above. Harding, Lindé, and Trabandt (2023) and Harding, Lindé, and Trabandt (2022) have shown that a nonlinear New Keynesian model with a nonlinear Phillips curve, in which the slope is state-dependent accounts much better for inflation dynamics in deep crisis than the linearized model. We contribute to this literature by introducing household heterogeneity in a model with a state-dependent nonlinear Phillips curve.

We include a state-dependent slope of the Phillips curve similar to Erceg, Jakab, and Lindé (2021) into an otherwise standard nonlinear HANK model, as in for instance Auclert, Rognlie, and Straub (2018). This model framework allows us to analyze the implications of a state-dependent Phillips curve slope on the propagation of demand and supply shocks in a heterogeneous agent environment. Importantly, our model allows for a two-way interaction between inflation and inequality during periods of inflation surges as well as during periods when inflation is persistently subdued.

Our results suggest that within our model framework inflationary pressures are amplified, while deflationary pressures are dampened. This is due to the state-dependency of the Phillips curve slope. This model feature allows us to match volatility and skewness from post-war US time series data on inflation and consumption growth. Our HANK environment allows us to study not only how supply and demand shocks propagate into inflation but also how they affect inequality. Our results suggest that inflationary cost-push shocks and contractionary demand shocks increase income inequality, because they imply an increase in the real interest rate. Higher real interest rates lead to an increase in capital income, which benefits savers with large asset holdings, while it negatively affects borrowers. The increase in inequality is amplified by the state-dependent Phillips curve slope. Conversely, the reduction in inequality in response to cost-pull or an expansionary demand shock is dampened in our setup, because real interest rates react less strongly to these shocks than in a model framework where the Phillips curve has a constant slope. Over the business cycle this result implies that inequality is skewed to the the right. Using US data on the standard deviation of log household income, we show that this positive skewness in inequality can be observed in the data as well. This implies that inequality increases more strongly in recessions than it falls in boom periods. A HANK model without a state-dependent Phillips curve slope is not able to replicate this feature observed in the data.

A growing strand of literature studies the relationship between inequality, inflation, and monetary policy. Auclert (2019), F. Bilbiie (2018) and Kaplan, Moll, and Violante (2018) show that inequality affects monetary policy transmission in a HANK environment. Auclert et al. (2023) analyze the effects of an energy price shock and corresponding monetary and fiscal policies using a HANK model. Auclert et al. (2023) find that in contrast to standard RANK models, energy price increases cause a recession in HANK models due to a reduction in real wages and wage inflation. In addition, some recent work focuses on the impact of inequality on optimal monetary policy in a HANK framework (Acharya, Challe, and Dogra (2023), Bhandari et al. (2021), McKay and Wolf (2022)). We contribute to this literature by explicitly allowing for a nonlinear Phillips curve in a HANK model.Importantly we focus on the ability of our model to account for the skewness observed in both inflation and income inequality in US data.

Our results are in line with existing empirical evidence on the impact of inflation and monetary policy on inequality. Coibion et al. (2017) find that income and consumption inequality in the US increases in response to contractionary monetary policy shocks. Recent empirical evidence by Pallotti et al. (2023) suggests that the recent inflationary shock has affected European households heterogeneously. More recent evidence in Del Canto et al. (2023) suggests that cost-push shocks widen the welfare distribution, while expansionary monetary policy shocks tighten it. Similar to Furceri, Loungani, and Zdzienicka (2018) we find an asymmetric response of monetary easing and tightening on income inequality.

The remainder of the paper is organized as follows. In section 2 we introduce our model and the state-dependent Phillips curve. Section 3 explains our model calibration. In section 4 we present our results. Section 5 documents the robustness of our results. Finally, section 6 concludes.

2 Model

This section introduces the nonlinear HANK model we use for our analysis. Our model set-up is based on the nonlinear version of the canonical HANK model presented by Auclert, Rognlie, and Straub (2018) and Auclert et al. (2021). Our model features Rotemberg pricing and a central bank setting a monetary policy rate. Additionally, we introduce a nonlinear Phillips curve with a state-dependent slope.

2.1 Households

Households derive utility from consumption $c_{i,t}$ and disutility from supplying labor $n_{i,t}$. Household i takes the following utility maximization problem:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{i,t}^{1+\nu}}{1+\nu} \right) \tag{1}$$

$$\text{s.t.} \quad c_{i,t} + a_{i,t} \leq (1+r_t)a_{i,t-1} + y_{i,t} \tag{2}$$

$$a_{i,t} \geq \underline{a}. \tag{3}$$

Here, $(1+r_t)a_{i,t-1}$ denotes revenues from holding assets, β denotes the discount factor, and φ the disutility from labor. Households can buy assets subject to a borrowing constraint given by equation (3). The asset stock is idiosyncratic and type-specific. The households' real wage income is given by:

$$y_{i,t} = w_t e_{i,t} n_{i,t}, \tag{4}$$

where w_t is the real wage and $e_{i,t}$ is idiosyncratic, type-specific productivity. There exist n_e idiosyncratic productivity states. Furthermore, following Auclert, Rognlie, and Straub (2018) we assume that all households are employed by a union, working the same amount of hours, which implies $n_{i,t} = n_t$.

2.2 Phillips curve

Following Auclert, Rognlie, and Straub (2018) we assume sticky wages but flexible prices in our model. In models with heterogeneous agents and nominal rigidities this assumption avoids countercyclical profits and thus large undesirable redistribution effects. Firms face quadratic nominal wage adjustment costs à la Rotemberg, governed by the adjustment cost parameter ϕ. The wage Philips curve is given by:

$$\pi_t^w(1+\pi_t^w) = \kappa_t n_t \left(\varphi n_t^{\nu} - \frac{\varepsilon - 1}{\varepsilon} w_t c_t^{-\sigma} \right) + \beta \mathbb{E}_t \left[\pi_{t+1}^w (1+\pi_{t+1}^w) \right] + \epsilon_t, \tag{5}$$

with ϵ_t as a cost-push shock, $\mu = \frac{\varepsilon}{\varepsilon-1}$ as the steady state wage markup, and κ_t the state-dependent slope parameter of the Phillips curve.

There is recent evidence that suggests that especially in high and low inflation scenarios a non-linear, 'banana-shaped', Phillips curve is necessary to explain the inflation developments in deep crises. Harding, Lindé, and Trabandt (2022) and Harding, Lindé, and Trabandt (2023) use a Kimball aggregator in a nonlinear New Keynesian model to explain the missing deflation puzzle and the post-covid inflation. Following Erceg, Jakab, and Lindé (2021) we introduce this state-dependency of the Phillips curve slope by assuming the slope parameter κ_t to take the following functional form:

$$\kappa_t = \kappa e^{\chi(y_t - \overline{y})} \tag{6}$$

with $\chi \geq 0$ specifying the curvature of the Phillips curve and $\kappa = \frac{\epsilon}{\phi}$.[1]

The functional form can be interpreted as follows, when the output is at its steady state value $\overline{y}$, the slope parameter κ_t becomes time-invariant. When output rises above steady state output, κ_t increases, therefore the Phillips Curve becomes steeper, accounting for the fact that wages are adjusted more strongly when the output gap is positive. When output falls below the steady state value κ_t decreases, and the Phillips curve becomes flatter.

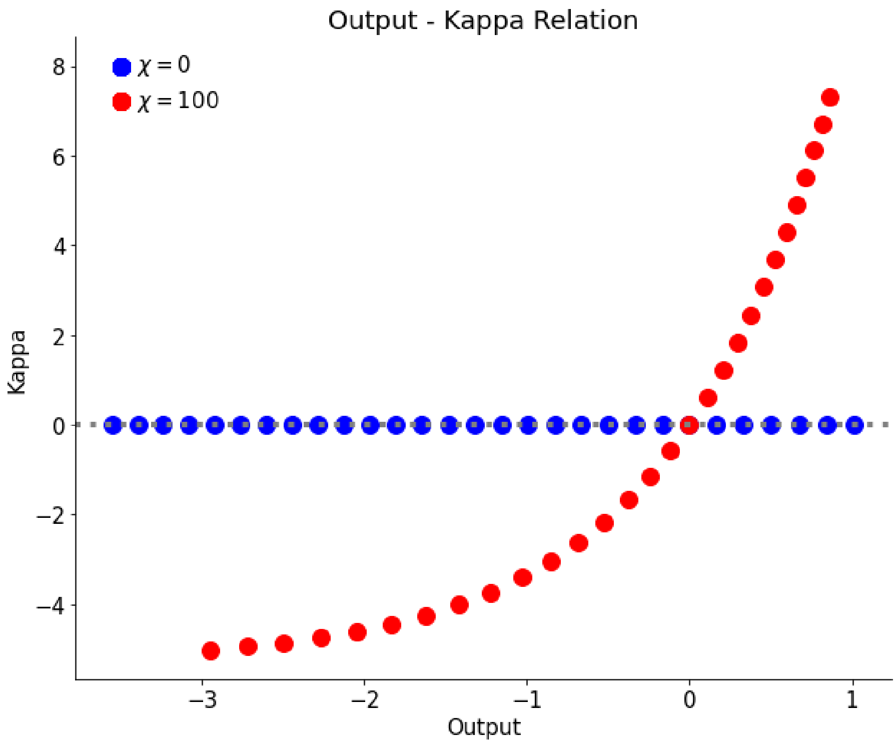

Figure 1: Relation between the Phillips curve parameter κ_t and the output gap following a demand shock

The introduction of a state-dependent κ_t allows us analyze the impact of very high and very low inflation on inequality taking into account the results by Harding, Lindé, and Trabandt (2023). Figure 1 displays the relationship between κ_t and the output gap in our model. When the output gap deviates negatively form the steady state, κ_t also shrinks relative to the steady state. When the output gap positively deviates from its steady state, κ_t increases compared to its steady-state value.

[1]We use this simple functional form for the curvature to make the analysis as transparent and tractable as possible. As a future extension we plan to consider a Kimball aggregator to microfound the shape of the Phillips curve as in Harding, Lindé, and Trabandt (2023).

2.3 Firms

The representative firm produces goods according to the following linear production function:

$$y_t = n_t. \tag{7}$$

Profit maximization under the assumption of flexible prices yields:

$$w_t = 1. \tag{8}$$

Goods inflation π_t is given by:

$$1 + \pi_t = (1 + \pi_t^w) \left(\frac{w_{t-1}}{w_t} \right). \tag{9}$$

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2.4 Monetary policy

We assume the central bank sets the nominal interest rate i_t, following a standard Taylor rule:

$$1 + i_t = ((1 + r)(1 + \pi))(1 + \pi_t)^{\phi_\pi} \left(\frac{y_t}{\tilde{y}_t} \right)^{\phi_y} e^{\gamma_t}, \tag{10}$$

where $\tilde{y}_t$ is potential output, which takes the value of steady-state output $\overline{y}$ in our model, r amnd π are the steady-state real interest rate and inflation rate, respectively, ϕ_π and ϕ_y the Taylor rule parameters on inflation and output, and γ_t is a monetary policy shock.[2]

2.5 Aggregation

The aggregate resource constraint is given by:

$$c_t = y_t \left(1 + \frac{\phi}{2} \left(\frac{W_t}{W_{t-1}} - 1 \right)^2 \right), \tag{11}$$

while the asset market clears at $a_t = 0$.

[2] Our Taylor Rule does not include a zero lower bound explicitly. However, due to the shape of the Phillips curve there is an endogenous lower bound that prevents negative policy rates in our simulations. In a future version of the paper we may consider to include an explicit zero lower bound. Considering the results of Fernández-Villaverde et al. (2023) it would be interesting to analyze how the zero lower bound and the state-dependency of the Phillips curve would interact.

2.6 Shocks

For the monetary policy shock γ_t and the cost-push shock ε_t, we specify the following AR(1) processes:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma, \tag{12}$$

$$\varepsilon_t = \rho_\epsilon \varepsilon_{t-1} + \epsilon_t^\varepsilon, \tag{13}$$

where $\epsilon_t^\gamma \sim \mathcal{N}(0, \sigma_\gamma^2)$ and $\epsilon_t^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ are exogenous shocks.

3 Calibration

We calibrate our HANK model to match seven moments of key macroeconomic quarterly US time series, the standard deviation, skewness and autocorrelation of inflation and consumption growth, and the correlation between inflation and consumption growth. To match the model moments as close as possible to the ones observed in the data, we select the following seven model parameters: $\{\beta, \phi, \rho_\gamma, \rho_\varepsilon, \sigma_\gamma, \sigma_\varepsilon, \chi\}$.

The discount factor is set to $\beta = 0.823$. This value is necessary to achieve a sufficiently wide asset distribution in our one-asset HANK model. The inverse elasticity of intertemporal substitution and the inverse Frisch elasticity are set to $\sigma = 2$ and $\nu = 2$. Further, the steady state wage mark-up is $\mu = 1.1$, resulting from a substitution elasticity between intermediate goods of $\varepsilon = 11$, which implies labor disutility of $\varphi = 1/\mu = 0.909$. The wage adjustment cost parameter is set to $\phi = 207.67$, such that it matches a Calvo wage stickiness of $\theta_w = 0.85$, implying an average price duration of approximately 1.5 years. The curvature parameter of the Phillips curve is set to $\chi = 100$. The Taylor rule parameters are set to standard values $\phi_\pi = 1.5$ and $\phi_y = 0.25$. The steady-state net inflation rate is $\pi = 0$ and steady-state output is $\overline{y} = 1$.

We introduce a monetary policy shock γ_t and a cost-push shock ε_t where the autocorrelation parameters are set to $\rho_\gamma = 0.9$ and $\rho_\varepsilon = 0.9$. Both shocks are exogenous with $\epsilon_t^\gamma \sim \mathcal{N}(0, \sigma_\gamma^2)$ and $\epsilon_t^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, where the standard deviations are set to $\sigma_\gamma = 0.01/45$ and $\sigma_\varepsilon = 0.02/21$.[3]

The heterogeneity in our model stems from heterogeneous idiosyncratic productivity states and heterogeneous asset holdings. There exist n_a idiosyncratic asset holding states, i.e. gridpoints on the asset grid, and n_e idiosyncratic productivity states. The number of gridpoints is set to $n_a = 500$ for the asset distribution and $n_e = 11$ for the productivity gird. We solve the nonlinear HANK model using the Sequence-Space Jacobian software package developed by Auclert et al. (2021). The model results in Table 1 and 6 are generated by a long model simulation over 1000 periods, where a random demand and

[3]In a future version we plan to consider other types of demand shocks, as for example consumption preference shocks or government spending shocks.

cost-push shock is drawn in every period. The periods in our model refer to quarters. In each simulation period, the algorithm solves a perfect foresight problem, where the economy is hit by an unexpected additional shock and agents do not anticipate any further shocks in future. We introduce a homotopy solver into the SSJ package to make the solution algorithm faster for long simulations.

3.1 Data

To match our model to US macro data, we use US data on annualized PCE inflation and real personal consumption expenditures excluding food and energy from 1967Q1 to 2019Q4. The evolution of quarterly inflation and consumption growth over the time period considered is shown in Figure 2.

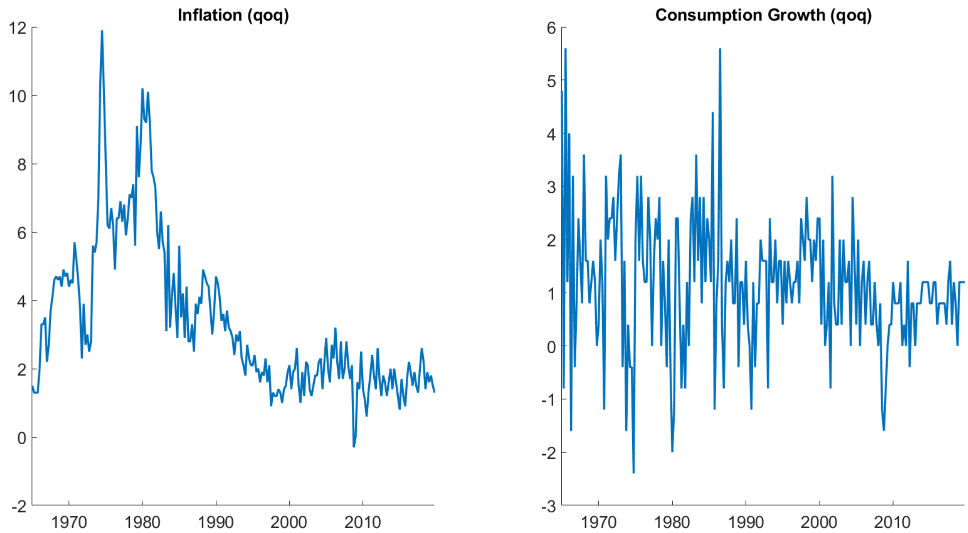

Figure 2: Quarterly US inflation (annualized) and consumption growth (annualized) time series 1965-2019.

To measure income inequality in the U.S., we construct an inequality measure using data on labor and capital income from the Current Population Survey (CPS). Following Heathcote et al. (2023) we rely on household data from the Annual Social and Economic (ASEC) supplement of the CPS from 1967 to 2019. We exclude households with zero or negative ASEC weight and households with no reference person or with no household member between 25 and 60 years of age. The income measure is constructed as the sum of labor income (WAGESALARY), capital income (CAPITAL_INC_NIPA) and income from self employment (SELF_EMP_INC_NIPA), divided by the number of adult equivalents in the household.

We use the standard deviation of log income to measure inequality in our sample. First, for each year in the sample, we discretize the income distribution using percentiles. We then take the logarithm of these percentiles and compute the weighted cross-sectional standard deviation in each year. Note that we exclude the lowest seven percentiles, because income is equal to zero for them in some years. To still properly represent the bottom income percentiles, the eighth percentiles is weighed by the factor eight, while all higher percentiles are weighed by the factor one. Following F. O. Bilbiie, Primiceri, and Tambalotti (2023) we detrend the resulting time series using a band-pass filter that extracts fluctuations with periodicities lower or equal to 30 years, because inequality shows a pronounced upward trend over the last 50 years that our business cycle model would not be able to match. Figure 3 shows the resulting filtered time series of inequality. We check the robustness of our detrending method by using different filters on our inequality time series in Appendix B.

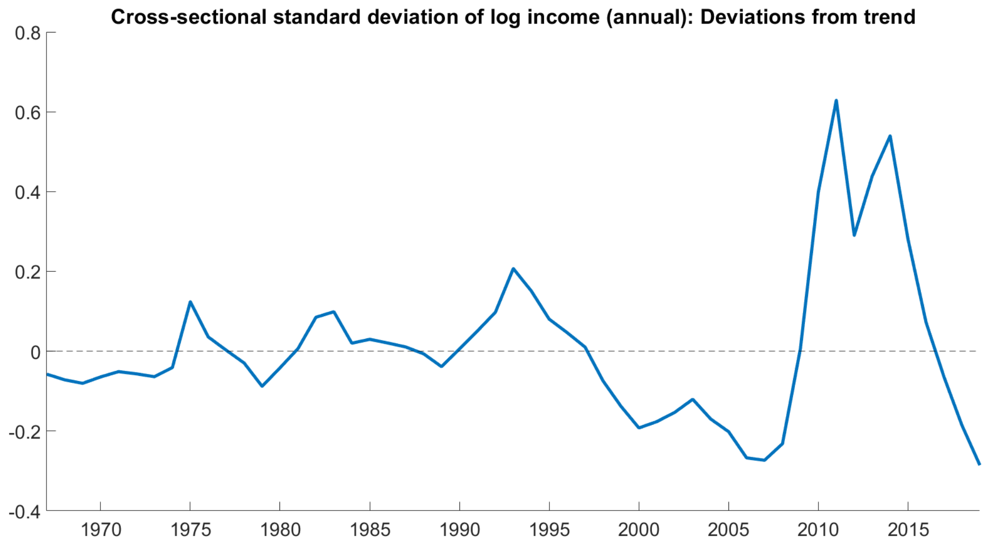

Figure 3: Cyclical fluctuations of log labor and capital income inequality.

4 Results

Table 1: Model vs. Data Statistics

	Model ($\chi = 100$)	Model ($\chi = 0$)	Data	95% confidence interval	
Standard deviation π_t	2.23	1.82	2.29	2.00	2.54
Skewness π_t	1.02	-0.09	1.24	0.93	1.55
Autocorrelation π_t	0.91	0.91	0.92	0.89	0.94
Standard deviation $\triangle y_t$	1.24	1.00	1.14	1.00	1.27
Skewness $\triangle y_t$	0.09	0.02	0.06	-0.45	0.61
Autocorrelation $\triangle y_t$	-0.01	-0.01	0.14	-0.01	0.29
Correlation π_t, $\triangle y_t$	-0.21	-0.21	-0.07	-0.24	0.11

Table 1 compares the results from a model simulation of randomly drawn demand and cost-push shocks over 1000 periods to the moments observed in the data. It shows that the nonlinear model with state-dependent Phillips curve matches the standard deviations, skewness and (auto-)correlations of inflation and consumption growth well. All of them lie within the 95 % confidence interval. In contrast, the model with a constant Phillips curve is not able to march especially the positive skewness of inflation.

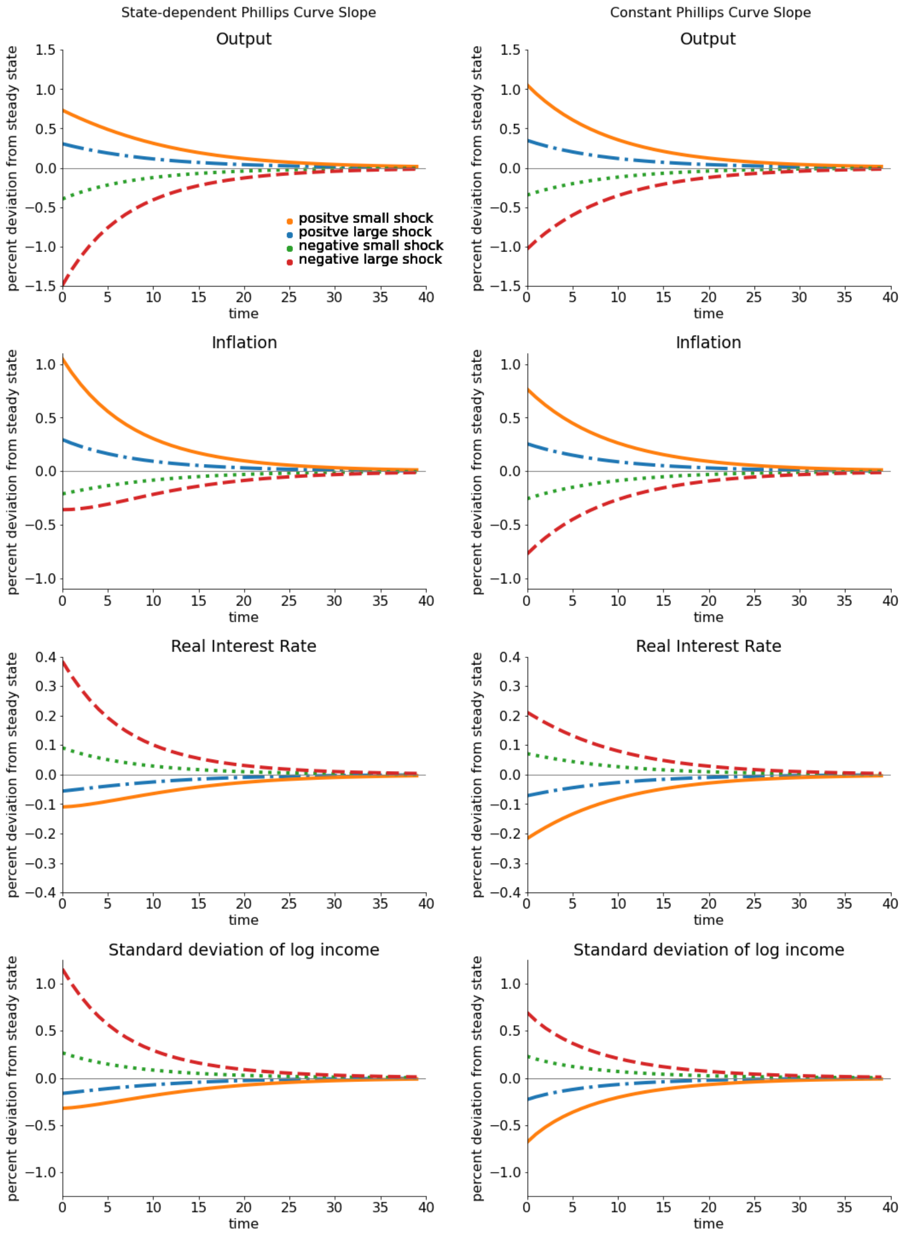

Figure 4: Impulse responses following a demand shock in a HANK model

Figure 4 shows the implications of a state-dependent Phillips curve slope for the propagation of demand shocks in the model. When the slope is constant, the impact of a demand shock is symmetric, meaning that an equal-sized positive and negative demand shock causes an equal-sized decrease and increase in output and inflation. Figure 4 shows that introducing a state-dependent slope parameter κ_t has no significant impact for small demand shocks, represented by the green dotted and blue dash-dotted line. This is due to the approximate linearity of the Phillips curve close to the steady state (see Figure 1). However, for larger shocks the responses following equal-sized positive vs. negative demand shocks become increasingly asymmetric. The intuition of the state-dependent Phillips curve slope is that wages are adjusted more strongly when the economy is in a boom, i.e. when the output gap is positive, which leads to higher inflation. This dampens the positive reaction of output following a positive demand shock. In a recession wages are not decreased as much as expected in a linear New Keynesian model, which dampens the drop in inflation. The decrease in output is however amplified as the central bank decreases the nominal interest rate less, as inflation falls less, and therefore the positive second-round effect of a lower nominal interest rate is smaller and output decreases more strongly.

Following a positive demand shock the increase in inflation is higher than the decrease in inflation following an equal-sized negative demand shock. The response of the standard deviation of log household income shows that this asymmetric response to shocks also carries over to our measure for inequality. Following a positive demand shock, output increases, the inflation rate increases and the real interest rate decreases. The shock affects agents differently. The labor income of hand-to-mouth consumers increases due to the increase in output. The same is true for the labor income of asset-holding agents. However, their additional asset income decreases due to the decrease of the real interest rate. This leads to a reduction in inequality. This reduction is dampened if a nonlinear Phillips curve with a state-dependent slope parameter is considered. Following the same positive demand shock, inflation increases more in this setting, which induces the central bank to increase the nominal interest rate more and therefore, the real interest rate decreases less. Consequently, the asset income decreases less than in the model with a constant Phillips curve slope. This means inequality falls less following a positive demand shock. Following a negative demand shock, output and inflation decrease and the real interest rate increases. Therefore, the labor income of hand-to-mouth consumers and asset holders decreases. The asset income of the asset holders increases due to the increase in the real interest rate. Therefore, inequality increases as the real interest rate increases. This effect is amplified by the state-dependent slope of the Phillips curve. Including a state-dependent Phillips curve the decrease in inflation following a negative demand shock is dampened, implying a stronger increase in the real interest rate. The resulting increase in asset-income causes inequality to raise more than in the model with constant Phillips curve slope.

Figure 5 shows that the state-dependent Phillips curve has similar implications for the response of the economy to a cost-push shock. The inflation response to an equal-sized positive supply shock is amplified, while the response to a negative shock is dampened.

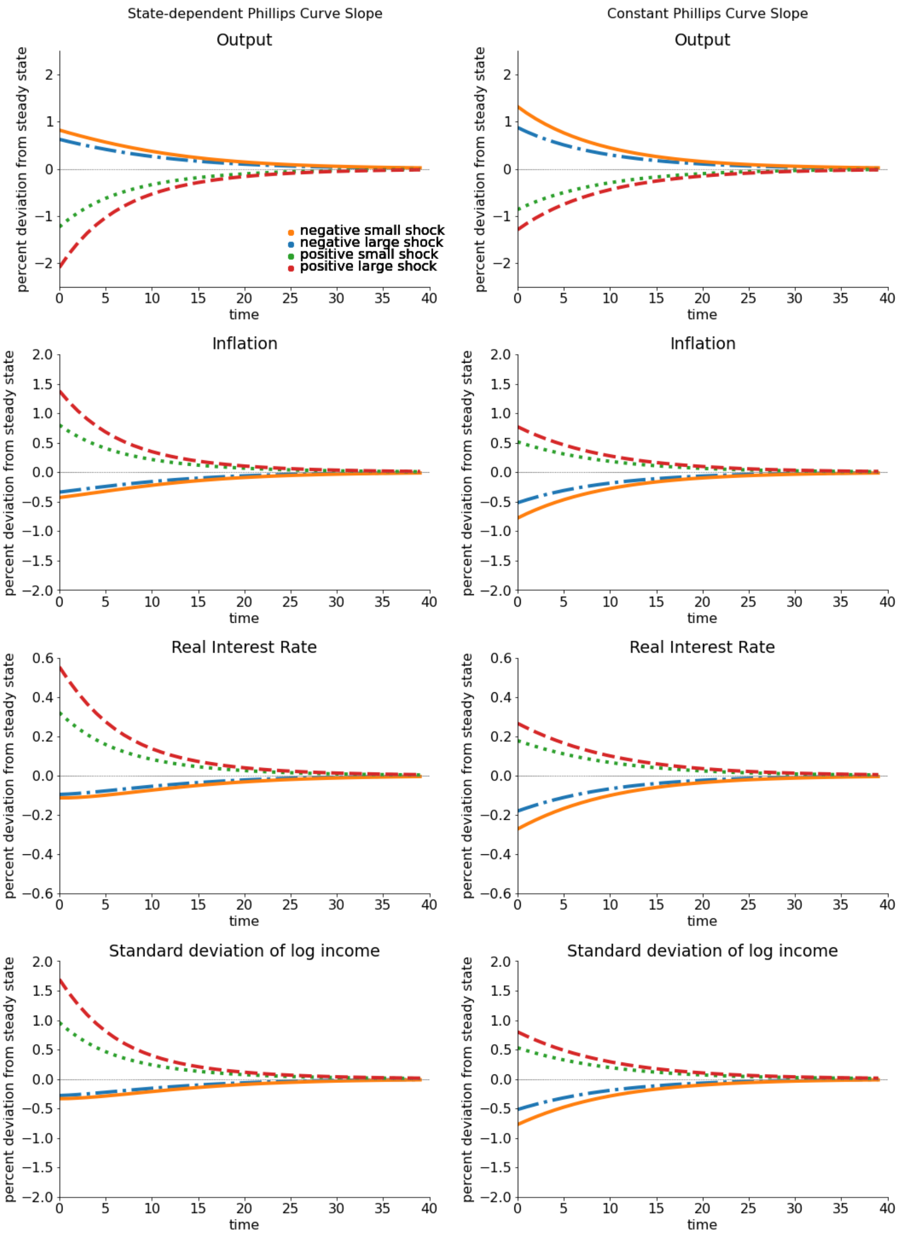

Figure 5: Impulse responses following a cost-push shock in a HANK model

Accordingly, output declines more strongly in response to a positive cost-push shock than it increases in response to a negative shock of the same size. Following a deflationary cost-pull shock, output increases and the inflation rate decreases. Due to the central bank's reaction to lower inflation, the real interest rate decreases as well. This means that the wage income of hand-to-mouth agents and asset holders increases. However, the asset income of asset holders decreases. Therefore, inequality decreases as well. This effect is smaller if a HANK model with a state-dependent Phillips curve slope is considered. Due to the slope of the Phillips curve the inflation rate decreases less and consequently the real interest rate decreases less as well. Compared to the constant Phillips curve slope the asset income of asset holders decreases less and therefore inequality decreases less.

For an inflationary cost-push shock the opposite is true. Output decreases as inflation rates increase and the real interest rate increases. In the nonlinear HANK model with a state-dependent Phillips curve the increase in inflation is amplified, therefore the increase in the real interest rate is larger as well. As a result, inequality increases more.

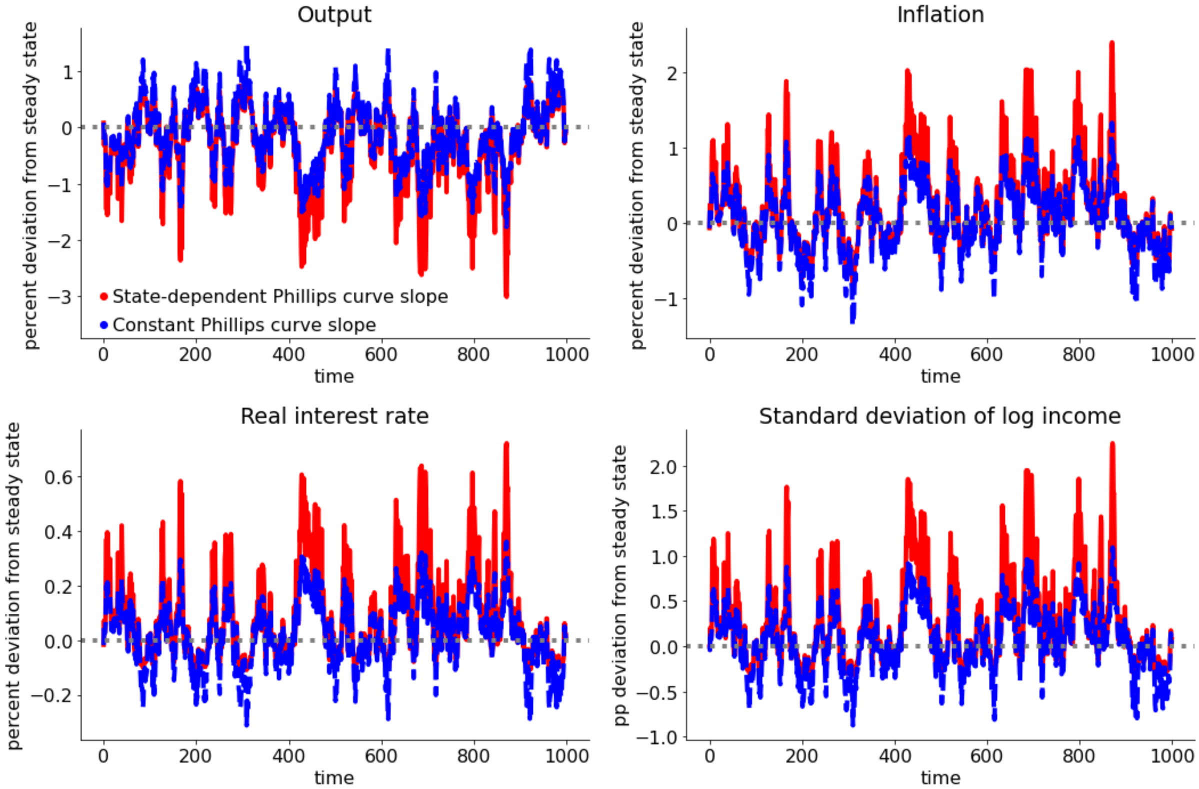

Figure 6: Long simulation in a HANK model over 1000 periods with demand and cost push shocks

To check whether the observed asymmetry in impulse responses prevails over a long time period, we simulate a sequence of 1000 randomly drawn demand and supply shocks. The standard deviations of the shocks are set to match long-term US data for inflation and consumption growth, as described in Section 3. Figure 6 shows that for the same draw of shocks, the positive inflation response is almost twice as large when the slope of the Phillips curve is state-dependent. In contrast, deflationary pressures are dampened in the nonlinear simulation, but the difference is less significant. The asymmetry of inflation impulse responses is more pronounced for larger shocks. For output, negative responses are much larger than positive ones, as implied by Figure 4 and 5. Figure 6 shows that over a long time period in which the economy is hit by supply and demand shocks, inequality tends to increase much more strongly than it decreases. The standard deviation of log household income increases by almost twice as much in the state-dependent Phillips curve slope model compared to a model including a constant Phillips curve, while the reductions in boom periods are dampened.

Table 2: Standard deviation of logs: Data vs. Model Statistics

	Model ($\chi = 100$)	Model ($\chi = 0$)	Data	95 % confidence interval	
Mean	0.95	0.95	1.06	0.99	1.14
Skewness	1.30	0.04	1.25	0.42	1.77
Standard Deviation	0.005	0.003	0.19	0.13	0.23

Data: Labor + capital income, US 1967-2021; Band-pass filter that extracts fluctuations with periodicities lower than 30 years.

All told, the reaction of inequality to shocks to the economy in this set-up depends strongly on the reaction of the real interest rate. In the nonlinear HANK model with a state-dependent Phillips curve parameter κ_t, the real interest rate is increasing much more than decreasing. Therefore, increases in inequality are amplified whereas decreases in inequality are dampened when a nonlinear state-dependent Phillips curve is considered. This implies that over the business cycle, inequality increases more strongly during recessions than it falls in boom periods.

Using our long-term simulation displayed in Figure 6 we can match the standard deviation of log household income of the US not only in the steady-state, but also the skewness of the standard deviation of log household income in the data, as shown in Table 2. Our model correctly predicts that the standard deviation of log household income ratio is positively skewed over time. A model with a constant Phillips curve slope predicts a skewness close to zero, which implies a symmetric response of inequality to positive and negative demand and supply shocks. However, we are not able to match the standard deviation of income inequality in our model. This is mainly due to the fact that our set-up is not able to produce the very large fluctuations we observe in the data. One of the main simplifications we use is, that the labor supply is provided by unions and uniform across agents. This means we are not able to capture unemployment as a source for inequality. Following Chang and Schorfheide (2023), unemployment seems to be a large channel, so adding unemployment may increase the volatility significantly. Additionally, our model features only liquid assets therefore the reactions of inequality are smaller as agents are either hand-to-mouth consumers or they are able to insure against their income risk. We are missing a very important group of agents the so-called wealthy hand-to-mouth which are holding illiquid assets, and therefore are restricted in their flexibility to react to negative shocks. As noted by Kaplan and Violante (2018) introducing an liquid as well as an illiquid asset state is crucial as is allows to feature less poor but instead more wealthy hand-to-mouth consumers that are responsive to changes in illiquid asset returns but not able to fully smooth consumption compared to a liquid asset holder. In a future version, we are planning to extend the model to also match the observed volatility of the standard deviation of logs.

5 Robustness

We examine the robustness of our results by checking how well our model matches the moments of alternative measures of inequality. Using the same dataset as described in section 3.1, we calculate the moments of the ratio between the top ninety (eighty) percent of and the bottom ten (twenty) percent of the income distribution. Table 3 shows that the model with state-dependent Phillips curve slope matches the skewness of the ninety-ten ratio found in the data, while the model with constant slope underestimates the skewness. Table 4 shows that the eighty-twenty ratio is also positively skewed, but to a smaller extent, because the tails of the income distribution are not considered within this measure. We are also able to match the positive skewness even though the data suggest a lower point estimate. Similar to the baseline analysis with the standard deviation of log income, the model is not able to generate sufficiently large fluctuations to match the standard deviation of both ratios in the data.

Table 3: Ninety-ten ratio: Data vs. Model Statistics

	Model ($\chi = 100$)	Model ($\chi = 0$)	Data	95% confidence interval	
Mean	10.83	10.83	16.36	14.65	18.24
Skewness	1.32	0.01	1.10	0.36	1.60
Standard Deviation	0.1	0.08	4.43	3.25	5.41

Data: Labor + capital income, US 1967-2019; Band-pass filter that extracts fluctuations with periodicities lower than 30 years.

Table 4: Eighty-twenty ratio: Data vs. Model Statistics

	Model ($\chi = 100$)	Model ($\chi = 0$)	Data	95% confidence interval	
Mean	3.3	3.3	4.27	4.10	4.45
Skewness	1.30	0.00	0.48	0.01	0.94
Standard Deviation	0.02	0.01	0.24	0.19	0.27

Data: Labor + capital income, US 1967-2019; Band-pass filter that extracts fluctuations with periodicities lower than 30 years.

6 Conclusion

We introduce a nonlinear Phillips curve with state-dependent slope into an otherwise standard one-asset HANK model. This allows us to match the positive skewness of inequality and inflation we observe in post-war US data. A version of our model with a constant Phillips curve slope is not able to match this feature in the data. We find that the increase in inequality in recessions is amplified in this set-up, while the inequality reduction in a boom is dampened. This is due to asymmetries in the response of the real interest rate to shocks in the economy. Since inflation tends to increase more strongly than it decreases, the same holds for the real interest rate and thus the asset income of asset holders. This result holds for supply and demand shocks in our model.

For future analysis it would also be interesting to relax the assumption that all agents work the same hours and thus earn the same wage. Additionally it will be interesting to introduce illiquid assets into the model framework to be able to incorporate the effects on e.g. wealthy hand-to-mouth consumers. Our versatile and simple model set-up has the potential to include extensions across several different topics. We are able to consider different monetary regimes, like average inflation targeting for instance, that could have an impact on inequality. In addition, the model could be extended along some dimensions. Optimal monetary policy analysis in this setting could provide interesting insights on whether the central bank should take the impact on inequality into account when battling high inflation. On the other hand we could include redistributive fiscal policies, for instance, that could dampen increases in inequality.

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A Derivation of wage Phillips curve

In this section, we derive the nonlinear wage Phillips curve given by 5. In our set-up, unions face quadratic nominal wage adjustment costs à la Rotemberg (1982). At time t, union j sets its wage $W_{j,t}$ to maximize the utility of its average worker. The maximization problem is defined as follows:

$$\max_{W_{j,t}} \sum_{s=0}^{\infty} \mathbb{E}_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{n_t^{1+\nu}}{1+\nu} - \frac{\phi}{2} \left(\frac{W_{j,t+s}}{W_{j,t+s-1}} - 1 \right)^2 \right], \tag{14}$$

$$\text{s.t.} \qquad n_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t. \tag{15}$$

Unions combine invidual labor into tasks, which face demand given by (15).

Using (15), household real earnings are defined as follows:

$$z_t = \int_0^1 \frac{W_{j,t}}{P_t} n_{j,t} dj = \frac{1}{P_t} \int_0^1 W_{j,t} \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t dj \tag{16}$$

We assume that all income from the union wage change is consumed immediately, which implies $\frac{\partial c_t}{\partial W_{j,t}} = \frac{\partial z_t}{\partial W_{j,t}}$ by the envelope theorem:

$$\frac{\partial c_t}{\partial W_{j,t}} = \frac{\partial z_t}{\partial W_{j,t}} = (1-\varepsilon) \frac{1}{P_t} \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t = (1-\varepsilon) \frac{1}{P_t} n_{j,t} \tag{17}$$

The derivative of hours worked by household i (from (15)) with respect to wage $W_{j,t}$ is given by:

$$\frac{\partial n_{i,t}}{\partial W_{j,t}} = -\varepsilon \frac{n_{j,t}}{W_{j,t}} \tag{18}$$

Using (17) and (18), we obtain the following first-order condition of the union:

$$c_t^{-\sigma} (1-\varepsilon) \frac{1}{P_t} n_{j,t} + \varepsilon \varphi n_t^{\nu} \frac{n_{j,t}}{W_{j,t}} - \phi \frac{1}{W_{j,t-1}} \left(\frac{W_{j,t}}{W_{j,t-1}} - 1 \right) + \beta \phi \mathbb{E}_t \frac{W_{j,t+1}}{W_{j,t}^2} \left(\frac{W_{j,t+1}}{W_{j,t}} - 1 \right) = 0 \tag{19}$$

In equilibrium all unions set the same wage, which implies $W_{j,t} = W_t$ and $n_{j,t} = n_t$:

$$c_t^{-\sigma} (1-\varepsilon) \frac{1}{P_t} n_t + \varepsilon \varphi n_t^{\nu} \frac{n_t}{W_t} - \phi \frac{1}{W_{t-1}} \left(\frac{W_t}{W_{t-1}} - 1 \right) + \beta \phi \mathbb{E}_t \frac{W_{t+1}}{W_t^2} \left(\frac{W_{t+1}}{W_t} - 1 \right) = 0 \tag{20}$$

Define wage inflation such that $\pi_t^W = \frac{W_t}{W_{t-1}} - 1$:

$$c_t^{-\sigma}(1-\varepsilon)\frac{1}{P_t}n_t + \varepsilon\varphi n_t^{\nu}\frac{n_t}{W_t} - \phi\frac{1}{W_{t-1}}\pi_t^W + \beta\phi\mathbb{E}_t\frac{1}{W_t}(\pi_{t+1}^W + 1)\pi_{t+1}^W = 0 \tag{21}$$

$$\Leftrightarrow \quad c_t^{-\sigma}(1-\varepsilon)w_t n_t + \varepsilon\varphi n_t^{\nu} n_t - \phi(\pi_t^W + 1)\pi_t^W + \beta\phi\mathbb{E}_t(\pi_{t+1}^W + 1)\pi_{t+1}^W = 0, \tag{22}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage.

Finally, this can be rearranged such that we obtain our nonlinear wage Phillips curve:

$$\pi_t^w(1+\pi_t^w) = \frac{\varepsilon}{\phi}n_t\left(\varphi n_t^{\nu} - \frac{\varepsilon-1}{\varepsilon}w_t c_t^{-\sigma}\right) + \beta\mathbb{E}_t\big[\pi_{t+1}^w(1+\pi_{t+1}^w)\big]. \tag{23}$$

In our model we then introduce the state-dependent slope parameter $\kappa_t = \frac{\varepsilon}{\phi}e^{\chi(y_t - \overline{y})}$ as described in section 2. Equation (23) represents the special case when output is at potential, i.e. $y_t = \overline{y}$.

B Robustness of de-trending method

Table 5: Data moments std of logs with different time series filters

	Band-pass 30y (baseline)	Band-pass 8y	HP filter	Hamilton
Skewness	1.24	0.43	1.00	2.16
SD	0.19	0.07	0.07	0.18

Table 5 presents the skewness and standard deviation of the standard deviation of log income in the dataset described in section 3.1 using four different de-trending methods to check the robustness of using the band-pass filter to extract fluctuations with periodicities lower than 30 years. First, we use a band-pass filter with an upper bound of 8 years, as often used for business cycle analysis. This yields a smaller, but still positive, skewness and a smaller standard deviation. Next, we use a two-sided Hodrick-Prescott (HP) filter, where we set the smoothing parameter to $\lambda = 6.25$, as is suggested for annual data by Ravn and Uhlig (2002). The results for the two-sided HP filter are close to our baseline results. Finally, we use a Hamilton filter (Hamilton (2018)), where we set the lead length to 2 and the lag length to 1, as recommended for annual data. The resulting cyclical component shows almost the same volatility as in the baseline analysis, but an even higher skewness. On average, all de-trending methods yield a skewness of 1.21, which is close to the value of 1.30 predicted by our model.