Forward Guidance and the Zero Lower Bound on Inflation Expectations*

Clara Lindemann**

Draft

November 6, 2023

Abstract

In this paper I explore unconventional monetary policy interventions as a potential cause for a zero lower bound on inflation expectations. I therefore implement a zero lower bound on inflation expectations and compare this to a situation where the central bank engages in unconventional monetary policy, especially forward guidance. My simulations show that in a New Keynesian model with a central bank that uses forward guidance policy a zero lower bound on inflation expectations can be a result of monetary policy.

JEL Classification: E31, E52, E58
Keywords: inflation expectations, forward guidance, zero lower bound

* I'm grateful for the comments and suggestions I received from colleagues, the participants of the Goethe University Macro Brown Bag Seminar and, especially for the feedback I received from my supervisor Prof. Dr. Mathias Trabandt.
** Goethe University Frankfurt am Main, c.lindemann@econ.uni-frankfurt.de

1 Introduction

The formation of inflation expectations and its reaction to changes in the economic situation is an important factor in the macro economy. Inflation expectations play a great role in the transmission mechanism of monetary policy. Precise modelling of them is crucial to deliver correct analysis of the effects of monetary policy and other shocks to the economy.

This paper's aim is to explore how a Zero Lower Bound (ZLB) on inflation expectations an empirical finding of Gorodnichenko and Sergeyev (2021), can be explained. Their analysis of surveys finds large asymmetries in household's inflation expectations. In particular, households on the one hand are overestimating inflation developments, but on the other hand do not expect negative inflation rates often enough. However, they do not elaborate further on the underlying causes of this observed ZLB on inflation expectations. Their finding of the ZLB on inflation expectations is most prominent in the 15 years following the financial crisis. This period coincides with a period of low interest rates in the US economy. I try to explain this observation by introducing unconventional monetary policy, especially forward guidance into a standard New Keynesian model. In the last decade the role and formation of inflation expectations has received a lot more attention. With their main conventional instrument, the policy rate, being constrained by the ZLB, central banks had to resort to unconventional policy measures to provide accommodation and fulfil their mandate. As inflation expectations of all agents play a major role in the effectiveness of these policies, a lower bound of inflation expectations would cause a major shift in the evaluation of the effectiveness of forward guidance as a monetary policy tool of the central bank. As the fundamental reason for this asymmetry in expectations strongly determines the effects on economic analysis, I explore forward guidance as one underlying cause for Gorodnichenko and Sergeyev (2021) observations.

I show that the phenomena modelled by them as a ZLB on inflation expectations can also be reached by a well designed forward guidance policy of the central bank. In this paper I create a baseline economy using a New Keynesian model proposed by Harding et al. (2022). Initially, I extend the model with a ZLB on inflation expectations as proposed by Gorodnichenko and Sergeyev (2021). Then I introduce a central bank that engages in threshold-based forward guidance. This shows that the observation of a ZLB on inflation expectations can be explained on forward guidance policies of the central bank.

As a robustness check I also use the Smets and Wouters (2007) model to confirm the findings of the main analysis in a workhorse macro model that includes more different shocks, additional frictions e.g. sticky wages and uses capital as a production factor.

This paper contributes to a long-standing literature on the formation of inflation expectations and also their role in the transmission of monetary policy. It builds upon contributions from Jonung (1981), Coibion et al. (2018), Gorodnichenko and Sergeyev (2021) and Coibion et al. (2021) amongst others.

The paper is organized as follows. I will first introduce the baseline model I use for all simulations and present an overview of the baseline results. In the third section I introduce a ZLB on inflation expectations and present the results compared to the baseline scenario. The fourth section presents the simulation results of an environment without a ZLB on inflation expectations, but instead a central bank that is very aware of agents inflation expectations and reacts to negative inflation expectations with forward guidance. The fifth section includes robustness check simulations in the Smets and Wouters (2007) model. Lastly, I discuss the findings and conclude. In the appendix, you can find more detailed information on the modeling of forward guidance decision of the central bank and robustness checks concerning the decision threshold of the central bank. I also provide the full set of log-linearized equations and the parameters values for the baseline model and the Smets and Wouters (2007) model in this paper.

2 Baseline Model

In the baseline model simulations, I use a simplified version of the linear New Keynesian model proposed by Harding et al. (2022).[1] It is a quarterly model featuring households, final and intermediate good firms, a government and a central bank which guides its policy according to a Taylor rule that includes a ZLB. This version of the model features sticky prices and Kimball aggregation in the goods market.[2]

2.1 Household

The representative household maximizes the utility function given by

$$\max_{(C_t, N_t, B_t)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - C\nu_t) - \frac{N_t^{1+\chi}}{1+\chi} \right), \tag{1}$$

with $0 < \beta < 1$.

[1]This paper focuses on the linear version of the model. An analysis taking into account the non-linear model and the potially interesting conclusions from that is left for future research.

[2]The full set of lienarized model equations and the paramterization used can be found in the appendix.

Following Harding et al. (2022) the households utility depends on the deviation of household consumption C_t from the consumption steady state level C, multiplied with an exogenous consumption preference shock ν_t, where $C\nu_t$ acts as a consumption reference level. The utility negatively depends on hours worked N_t. The utility is maximized subject to the budget constraint given by,

$$P_t C_t + B_t = (1 - \tau_N) W_t N_t + (1 + i_{t-1}) B_{t-1} - T_t + \Gamma_t \tag{2}$$

which states that the consumption expenditures and the purchases of government bonds B_t equals the households disposable income. This is the sum of after-tax labor income $(1 - \tau_N) W_t N_t$, its share of the intermediate firms' profits Γ_t, net of the households lump-sum tax payment T_t.

Utility maximization and log-linearization yields the following linear Euler equation:

$$0 = E_t \left[\breve{i}_t - \hat{\Pi}_{t+1} - \frac{\hat{c}_{t+1} - \breve{\nu}_{t+1}}{1 - \nu} + \frac{\hat{c}_t - \breve{\nu}_t}{1 - \nu} \right], \tag{3}$$

with $\breve{i}_t$ being the nominal interest rate and $E_t \hat{\Pi}_{t+1}$ the expected inflation rate in $t + 1$.[3]

2.2 Firms

2.2.1 Final Goods Production

There is a single final good Y_t produced from a continuum of differentiated intermediate goods $Y_t(f)$. The final goods producer face the following maximization problem:

$$\max_{Y_t, Y_t(f)} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df, \tag{4}$$

subject to the technology used to transforming the intermediate goods into final goods, following Kimball (1995):

$$\int_0^1 G\left(\frac{Y_t(f)}{Y_t}\right) df = 1, \tag{5}$$

$G(.)$ is given by the following strictly concave and increasing function:

$$G\left(\frac{Y_t(f)}{Y_t}\right) = \frac{\omega}{1 + \psi} \left((1 + \psi) \frac{Y_t(f)}{Y_t - \psi} \right)^{\frac{1}{\omega}} - \left(\frac{\omega}{1 + \psi} - 1 \right), \tag{6}$$

with $\omega = \frac{\phi(1+\psi)}{1+\phi\psi}$.

[3] Hat variables describe percent deviations from steady state. Breve variables describe absolute deviations from steady state.

ϕ describes the gross markup of the intermediate good firms and ψ controls the curvature of the intermediate firm's demand curve. If $\psi = 0$, the demand curve behaves as in the standard Dixit-Stiglitz aggregator. With $\psi < 0$ intermediate firms adjust their prices by less to a given change in marginal cost.[4]

2.2.2 Intermediate goods production

Each intermediate goods firm produces a single differentiated good from a continuum of intermediate goods $Y_t(f)$, $f \in [0, 1]$. It faces a varying demand from the final goods producing firms, depending on output price $P_t(f)$ and aggregate demand Y_t.
The production of the intermediate good is given by:

$$Y_t(f) = K(f)^{\alpha} N_t(f)^{1-\alpha}, \tag{7}$$

with capital K assumed to be fixed.

The intermediate goods price is determined by staggered nominal contracts as in Calvo (1983). In each period firm f can re-optimize its price $P_t(f)$ with probability $1 - \xi$. The probability of being able to adjust the price is constant over time and is independent of the time since its last price change. If the firm is not able to adjust the price in the given period the price is set as:

$$\tilde{P}_t = (1 + \pi) P_{t-1}, \tag{8}$$

with the steady-state net inflation rate π and the updated price $\tilde{P}_t$. For firms which are able to re-set the price $P_t^{opt}(f)$ the optimization problem is the following:

$$\max_{P_t^{opt}(f)} E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Theta_{t,t+j} \left((1+\pi)^j P_t^{opt}(f) - MC_{t+j}\right) Y_{t+j}(f), \tag{9}$$

with $\Theta_{t,t+j}$ being the stochastic discount factor and $Y_{t+j}(f)$ the demand from final goods firms given by their maximization problem.

2.3 Monetary and Fiscal Policy

The central bank sets the nominal interest rate using the following Taylor rule:

$$1 + i_t = max\left(1, (1+i)\left(\frac{1+\pi_t}{1+\pi}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_t^{pot}}\right)^{\gamma_x}\right), \tag{10}$$

[4]Note that the Kimball (1995) and the Dixit and Stiglitz (1977) aggregator can be considered observationally equivalent in a linearized model(Lindé and Trabandt (2018)).

with Y_t^{pot} being the level of output with flexible prices and i the steady state net nominal interest rate which is equal to $r + \pi$, with $r \equiv 1/\beta - 1$.

The government budget constraint is given by:

$$B_t = (1 + i_{t-1})B_{t-1} + P_t G_t - \tau_N W_t N t - T t, \tag{11}$$

where G_t are the real government expenditures on Y_t. Government debt as a share of nominal steady state GDP: $b_t = \frac{B_t}{P_t Y}$ is stabilized by a net lump-sum tax, $\tau = \frac{T_t}{P_t Y}$:

$$\tau_t - \tau = \varphi(b_{t-1} - b), \tag{12}$$

with τ and b, the steady state of τ_t and b_t.

2.4 Aggregate Resources

Aggregate output Y_t is give by:

$$Y_t = (p_t^*)^{-1} K^\alpha N_t^{1-\alpha}, \tag{13}$$

$$p_t^* = \int_0^1 \frac{1}{1+\psi} \left(\left(\frac{P_t(f)}{P_t} \frac{1}{\Lambda} \right)^{\frac{\phi}{1-\phi}(1+\psi)} + \psi \right) df, \tag{14}$$

with Λ as the Lagrangian multiplier and p_t^* being the aggregate price distortion term, this term vanishes when the model is linearized. Thus, in the linear model the aggregate output is given by:

$$C_t + G_t = K^\alpha N_t^{1-\alpha}. \tag{15}$$

2.5 Parametrization

I follow the parametrization proposed by Lindé and Trabandt (2018): the discount factor β is set to 0.995 , the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\chi} = 0.4$. The steady state net inflation rate $\pi = 0.005$, which implies a nominal interest rate $i = 0.01$. This implies an annualized nominal interest rate of 4%. The sensitivity of prices to marginal cost is determined by the gross markup $\phi = 1.1$, the stickiness parameter $\xi = 0.667$ and the Kimball parameter,$\psi = -12.2$. The Taylor rule parameters are set to the standard parameters $\gamma_\pi = 1.5$ and $\gamma_x = 0.125$.

The consumption preference shock used for the simulations follows an AR(1) process given by:

$$\nu_t - \nu = \rho_\nu(\nu_{t-1} - \nu) + \epsilon_{\nu,t}, \tag{16}$$

with $\rho_\nu = 0.95$.

2.6 Solving the Model

To perform the following simulations, I implement the set of linearized equations in Dynare.[5] Dynare is a software using MATLAB routines to solve and simulate linear and non-linear models including forward-looking variables. For the simulations, I use the perfect foresight solution algorithm using the simul command. More details on the implementation can be found in Juillard (1996). With this solution method, it is straightforward to implement a ZLB, as the solver can handle max operators within model equations. For the following simulations, I construct a baseline economy over 2000 quarters which is hit by a consumption demand shock every quarter. The shocks are drawn from a normal distribution calibrated so that the economy is bound to the ZLB in about 10 percent of all periods. Therefore the standard deviation of the shock is set to $\frac{1}{37}$. If the economy is at the ZLB the mean duration is 7.8 periods, the maximum duration at the ZLB is 26 periods, which equals 6.5 years.

In the baseline simulation, the median nominal interest rate is 4.4 percent and the median output gap is 0.05. The median inflation rate is 2.24 percent and the median expected inflation rate is 2.22 percent. Therefore, the implementation of a ZLB on inflation expectation will have an effect on the inflation expectation rates and hence, on realized inflation rates.

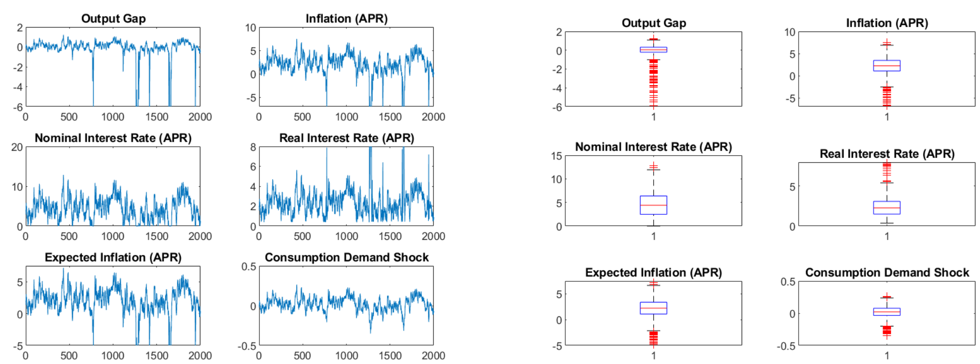

Figure 1: Results of the Baseline Simulations
The right chart shows the results over the timeline of 2000 periods. The left chart shows the same results in a box plot to show the distribution.

[5]More information on Dynare can be found here: Adjemian et al. (2022).

Figure 1 displays the results of the baseline simulation on the timeline and as a boxplot. The timeline shows that during these 2000 quarters the nominal interest rate is bound to the ZLB several times when the economy experiences a deep recession. This is also noticeable in the sharp drops in the output gap coinciding with large negative consumption demand shocks. The time and the boxplot both show several periods with inflation expectations below zero. Note, that the 10th percentile of inflation rate and expected inflation rate are below zero.

In 9.7 percent of the time the nominal interest rate is bound at the ZLB and the inflation expectations are below zero as well. This ensures that in the model specification with forward guidance the central bank will intervene to increase inflation expectations.

3 Zero Lower Bound on Inflation Expectations

In recent work, Gorodnichenko and Sergeyev (2021) suggest that there may be a ZLB on inflation expectations of households. This phenomenon may also apply to firms. However, there is no clear evidence on that. They use survey data from Japan and the US to document a floor to inflation expectations. Especially for Japan, they document that even though between 2004 and 2020 the realized inflation rate was below zero for 40 percent of the time, only 5 percent of the households had inflation expectations below zero. However, Gorodnichenko and Sergeyev (2021) also document that for example in the Great Recession the share of households that expect inflation rates equal to and below zero increased, such that less than 50 percent expected inflation above zero. They explore the consequences of a zero lower bound on inflation expectations on the effectiveness of monetary and fiscal policy of economies in a liquidity trap. Their analysis concludes that introducing such a lower bound has severe impacts on the effectiveness of monetary and fiscal measures. They find fiscal multipliers becoming finite and the impact of unconventional monetary policy measures weaker in an economy including a zero lower bound on inflation expectations.

3.1 Model with ZLB on Inflation Expectations

To include the ZLB on inflation expectations in the baseline model from Harding et al. (2022) used here, I define inflation expectations explicitly as a variable in the model.

In the log-linearized version of the model the Philips curve is given as:

$$\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{1}{1-\phi\psi} \hat{mc}_t. \tag{17}$$

To introduce the ZLB on inflation expectations I include $\tilde{E}_t \hat{\Pi}_{t+1}$ which describes the inflation expectations of the agents bound to zero:

$$\tilde{E}_t \hat{\Pi}_{t+1} = max(-\pi, E_t \hat{\Pi}_{t+1}); \tag{18}$$

with π being the steady state inflation rate. The new Philips Curve is now:

$$\hat{\Pi}_t = \beta\, max(-\pi, E_t \hat{\Pi}_{t+1}) + \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{1}{1-\phi\psi} \hat{mc}_t. \tag{19}$$

Within the perfect foresight simulation, the implementation is possible without any further adjustments to the model. This allows for a direct comparison of the baseline model calibration with the model specification including the ZLB on inflation expectations.

3.2 Results

In the baseline scenario, the expected inflation rate is below zero in 9.7 percent of the time. So the ZLB on inflation expectations will be binding in these periods. I find that the ZLB on inflation expectations does not influence the economy as long as inflation expectations do not fall below zero. Figure 2 shows a stylized example of the reaction of the economy to a negative demand shock with and without a ZLB on inflation expectations. This strong negative demand shock is followed by a decrease in consumption, an opening up of the output gap and a sharp drop in inflation. Therefore, the central bank reacts by cutting the nominal interest rate, which is bound to the ZLB.

Compared to this the reaction of inflation expectations and also realized inflation is limited in the second case. As inflation expectations are bound to zero the reaction of the central bank in cutting the inflation rate is also less negative. This limits the reaction of the nominal interest rate. Also, the reaction of consumption is less negative with the ZLB on inflation, as agents are not anticipating inflation to fall below zero. Therefore, the reaction of the output gap is also slightly smaller than in the baseline simulation. This effect becomes even more prominent when the unanticipated drop in inflation expectations is larger.

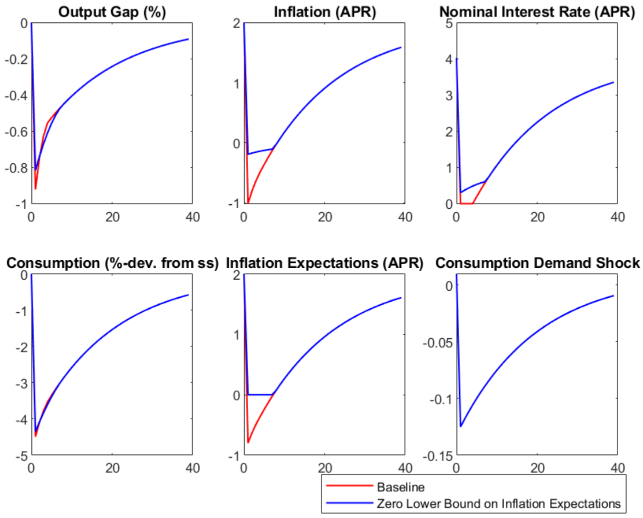

Figure 2: Comparison baseline vs. ZLB on inflation expectations
Impulse response function following a negative consumption demand shock in the baseline model and in the model including a ZLB on inflation expectations.

Considering a long time horizon, the ZLB on inflation expectations also acts as a lower bound on inflation. The ZLB on inflation expectations only influences the left side of the distribution, meaning it moderates the impact of negative consumption demand shocks on inflation rates. The 5th percentile of inflation is shifted to -0.2 percent from -1.1 percent in the baseline scenario. The same applies to the 5th percentile of the output gap it increases from -1.0 to -0.85, following the implementation of the inflation expectations ZLB. Moreover, the percentage of periods the economy is stuck at the ZLB is reduced to 6.4 percent from 10.3 percent.

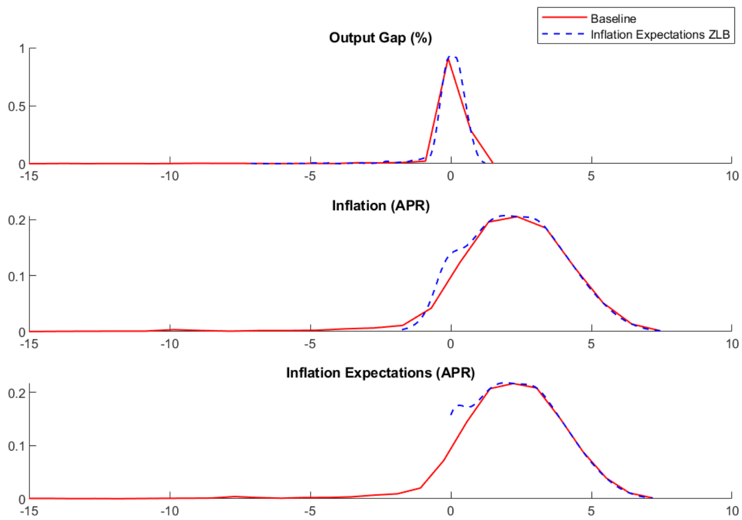

Figure 3: Density function: baseline simulation vs. ZLB on inflation expectations

Figure 3 shows the changes in distribution of the output gap, inflation, and inflation expectations from introducing the ZLB on inflation expectations. Inflation expectations are bound to zero and the share of periods in which inflation expectations are zero increases sharply. The inflation rate is also bound to a minimum value of -1 percent, following the introduction of a ZLB on inflation expectations. The share of periods with inflation rates around zero increases compared to the baseline simulation. For the output gap, there is a slightly larger share of periods with a small positive output gap, but less periods with a negative output gap are observed. In a situation with the economy above the steady state the ZLB on inflation expectations introduces no changes to the distribution.

4 Forward Guidance

Forward guidance is a monetary policy intervention where the central bank commits to a lower interest rate or an interest rate at the ZLB until there is an improvement of the economy or the objective of the central bank is met. As noted by Svensson (2014) there are central banks, such as the Reserve Bank of New Zealand or Norges Bank, that include forward guidance as an instrument in their standard monetary policy toolkit. However, other central banks such as the ECB, FED or Bank of England have introduced forward guidance as part of their monetary policy toolkit only when the nominal interest rate reaches the zero lower bound. In this case, forward guidance provides another monetary policy option when the nominal interest rate is bound to zero.

There are different ways implementing these policies. One possibility to implement a forward guidance strategy is to publish an expected interest rate path. This is an implementation technique used e.g. by the Reserve Bank of New Zealand (Svensson (2014)). An alternative strategy is to announce at which state of the economy a rise of the inflation rate will be possible again. This was a strategy the FED pursued in 2012 when they committed to a zero nominal interest rate as long as inflation remains below 2.5 percent and unemployment remains above 6.5 percent. As noted by Christiano et al. (2015) in this particular case the FED did not commit to taking action thereafter. This forward guidance technique is what I will refer to as threshold-based forward guidance, where the central bank commits to a threshold that needs to be reached for the central bank to return to its normal monetary policy strategy.

4.1 Implementation of Forward Guidance

The aim is to explain the findings from Gorodnichenko and Sergeyev (2021) with forward guidance interventions from the central bank. To achieve this, I allow the central bank, in a situation stuck at the ZLB, to choose the optimal duration of forward guidance so that inflation expectations do not fall below zero. Forward guidance here means the central bank announces to set the nominal interest rate to zero for a certain number of periods longer than the economy would be stuck at the ZLB without further intervention. In this implementation forward guidance is a monetary policy instrument that is only used when the nominal interest rate is stuck at the ZLB. Otherwise, the central bank uses a Taylor rule to conduct monetary policy. I use threshold-based forward guidance in these model simulations. Meaning that the central bank commits to implementing forward guidance until inflation expectations are above zero. The central bank always sets the minimum duration of forward guidance needed to bring inflation expectations above zero.

Within the model simulation forward guidance is implemented as follows: in each period the economy is hit by a randomly drawn consumption demand shock equal to the baseline scenario. If the economy is not bound to the ZLB the next period is simulated. Otherwise, the economy is bound at the ZLB of the nominal interest rate and the central bank checks whether the ZLB on expected inflation is binding, too. If this is the case the central bank engages in forward guidance for as many periods as needed to bring inflation expectations above zero. How much forward guidance is needed is determined endogenously and is depending on the drop in inflation expectations.
Over all 2000 simulation periods in 9.7 percent the ZLB is binding and inflation expectations are also below zero so that the central bank steps in and uses forward guidance. In the median, there are 9 periods of forward guidance needed if the central bank engages in forward guidance.

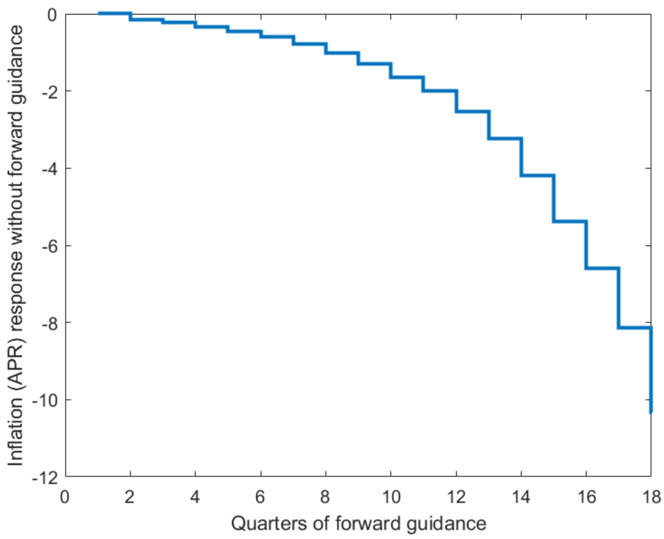

Figure 4: Quarters of forward guidance needed to increase inflation expectations above zero, depending on the depth of the inflation response without central bank intervention. The value is the lowest drop that was recorded for the number of quarters

Figure 4 shows very impressively the number of quarters that are needed to increase inflation expectations above zero. In particular, the number of quarters needed changes non-linear in the size of the drop. For small drops below zero it increases fast whereas the increase slows down for larger drops below zero. To counter a drop to -1 percent in inflation forward guidance for 8 periods is needed. However, to counter a drop to -5 percent inflation 15 quarters of forward guidance are needed. Following this analysis' the central bank needs to step in very determinedly to prevent a drop in inflation expectations below zero.

4.2 Forward Guidance Compared to ZLB on Inflation Expectations

In this experiment, the expected inflation rate is not mechanically restricted to zero, but the central bank uses forward guidance as an additional monetary policy tool to keep inflation expectations above zero.

The simulation results show that the forward guidance strategy is equally successful in bringing the expected inflation rate above zero as the ZLB on inflation expectations. In the forward guidance scenario, the central bank uses its announcements on future nominal interest rates to change agents' expectations about inflation developments. The central bank credibly commits to keeping interest rates low, which given everything else equal, increases inflation expectations. This effect passes through to inflation rates and improves the overall economic situation.

Table 1 shows that implementing a ZLB on inflation expectations or including a central bank using threshold-based forward guidance results in a comparable distribution of inflation expectations, inflation rate and the output gap. The median value for inflation expectations, inflation and the output gap are between 0.1 to 0.2 percentage points above the calibrated steady state value, i.e. 2 percent for inflation and inflation expectations and 0 for the output gap. This result is expected as both discussed interventions have a positive effect on the economic situation. Overall, median inflation expectations, inflation and the output gap of both scenarios are very much alike. As expected the distribution above the 50^{th} percentile is identical as both the ZLB and forward guidance only have an effect following a negative shock to the economy. Only a small difference is noticeable considering the 5^{th} percentile. In these deep recessions the forward guidance strategy results in a little more positive results on inflation expectation, inflation and the output gap. A main reason for the more positive outcome is that the central bank will always announce an additional quarter of forward guidance. This is caused by the quarterly nature of the model and can be observed in the inflation expectations distribution. Even if the inflation expectations drop just below zero, the central bank will always announce an additional full quarter of forward guidance. This means that the intervention from the central bank will result in a stronger positive effect on inflation expectations, and therefore also on inflation and output. This is well documented in figure 5 as the mechanical ZLB on inflation expectations binds the inflation expectations at zero. In contrast to that, the forward guidance strategy intervenes until the inflation expectations are above zero and which leads to inflation rates being potentially higher than with a ZLB. This problem can be alleviated by using a model which allows for more frequent changes of the monetary policy rate.

	Percentile	5^{th}	25^{th}	50^{th}	75^{th}	95^{th}
Inflation Expectations (APR)	Infl. Exp. ZLB	0	1.1	2.2	3.4	5.0
	Forward Guidance	0.1	1.2	2.3	3.4	5.0
Inflation (APR)	Infl. Exp. ZLB	-0.2	1.1	2.2	3.5	5.1
	Forward Guidance	0.1	1.2	2.3	3.5	5.2
Output Gap	Infl. Exp. ZLB	-0.8	-0.2	0.1	0.3	0.7
	Forward Guidance	-0.4	-0.1	0.1	0.3	0.8

Table 1: Percentile Comparison between Forward Guidance and ZLB on Inflation Expectations

Simulation results of inflation expectations (APR), inflation (APR) and output gap for an economy with a ZLB on inflation expectations and for an economy with a central bank that engages in forward guidance.

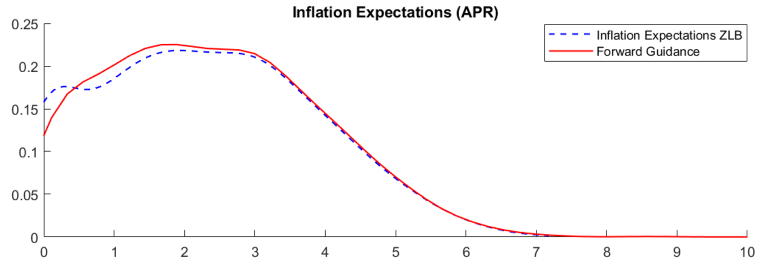

Figure 5: Forward Guidance vs. ZLB on inflation expectations
Density plots showing the results of forward guidance compared to inflation expectations ZLB.

Figure 5 documents that a properly tailored forward guidance strategy is capable of delivering the results on inflation expectations Gorodnichenko and Sergeyev (2021) use to justify the introduction of a new ZLB. These results show that forward guidance could be a potential cause for the observations of Gorodnichenko and Sergeyev (2021) if central banks, in their aim to raise inflation rates, influenced inflation expectations as well. It demonstrates that a tailored forward guidance strategy can explain the findings of Gorodnichenko and Sergeyev (2021) regarding the floor to consumers' inflation expectations very well. In a scenario with a central bank equipped with the tool of forward guidance, these results can be achieved in a standard New Keynesian model with rational agents. In the appendix, you can find an analysis including a central bank that reacts later to decreases in inflation expectations at the Zero Lower Bound. This shows that similar results can be reached and the exact threshold at which the central bank intervenes is somewhat flexible.

5 Robustness Checks

In this section, I want to examine the robustness of the results presented in section 4. In their first analysis Gorodnichenko and Sergeyev (2021) use a simple three equation New Keynesian model to introduce their idea. In this paper I analyze the reasons behind their findings using a simplified version of the Harding et al. (2022) model. To provide more evidence that my findings are independent of the exact model used but applicable to many New Keynesian models, I test the hypothesis that the ZLB on inflation expectations can be explained by threshold-based forward guidance of the central bank using the workhorse Smets and Wouters (2007) model.

5.1 Smets and Wouters (2007)

5.1.1 Model and Baseline Simulations

For this robustness analysis I use the Smets and Wouters (2007) model. Compared to the baseline model it also features capital and investment. It also includes sticky wages and more real rigidities as for example habit formation in consumption, adjustment cost of investments, variable capital utilization and fixed costs in production. Additionally, the model features seven different shocks influencing the dynamics of the economy. I use the risk premium shock so that the result of the baseline simulation has overall similar effects on the economy compared to the baseline model described in Section 2. It acts like a demand shock, meaning it has a positive effect on output, hours worked, inflation and nominal interest rate. As the consumption demand shock described in section 2, the risk premium shock affects the economy through the consumption euler equation. As mentioned by Smets and Wouters (2007) the variation of the nominal interest rate in the short and medium run is mainly explained by the demand and productivity shocks, especially the risk premium shock. Therefore, it is a good fit as this paper analyses central bank reaction to inflation changes and it shows that using the risk premium shock makes the analysis applicable to many situations.

The Smets and Wouters (2007) model features a central bank that follows a Taylor rule which resets the nominal policy rate in response to inflation and the output gap. Additionally, the central bank engages in interest rate smoothing. This means the nominal policy rate is adjusted gradually to changes in inflation and the output gap. To be able to implement a ZLB while keeping the interest rate smoothing feature of the Taylor rule I introduce a shadow nominal interest rate, R_0, which equals the unconstrained Taylor rule. The nominal policy rate, R, is set to zero if the shadow rate R_0 is below zero. Otherwise the nominal policy rate is equal to the shadow nominal policy rate. My simulations are based on the dynare code provided by Pfeiffer (2020) which is replicating the Smets and Wouters (2007) model, but is adapted so that it runs with dynare version 4.5. For my baseline simulations I keep the priors and the calibrated parameters according to the Smets and Wouters (2007) model. The Taylor rule is adjusted as described above and the variance of the shock is calibrated so that the economy is at the ZLB in 10 percent of the time. As the Smets and Wouters (2007) model incorporates more nominal rigidities than the Harding et al. (2022) model, therefore the risk premium shock is associated with a larger drop in the output gap than a similar shock in the Harding et al. (2022) model.

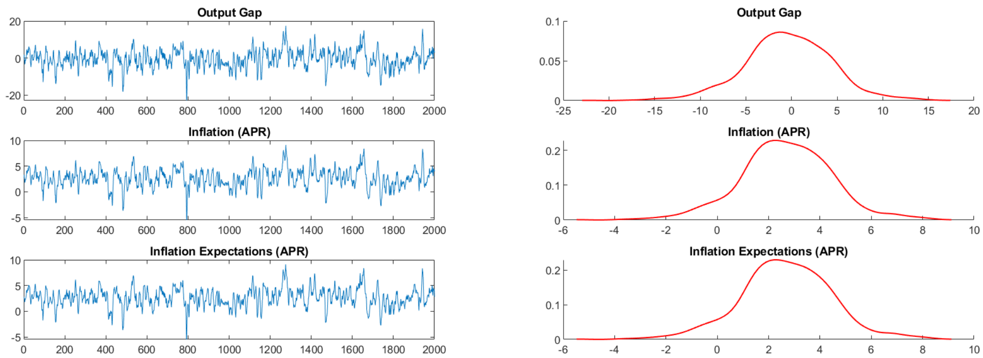

Figure 6: Results Baseline Simulations Smets and Wouters (2007) model
The right chart shows the results over the timeline of 2000 periods. The left chart shows the same results in a density figure to show the distribution.

Figure 6 shows the results from the baseline simulations using the Smets and Wouters (2007) model. During the 2000 period simulation, the economy is hit by a randomly drawn risk premium shock each quarter. The economy experiences several deep recessions with inflation expectations dropping well below several times. The density plot shows that there is a non-negligible number of periods where inflation expectations are well below zero. Therefore, implementing a zero lower bound on inflation expectations will impact the overall outcomes as inflation expectations impact the economic decisions of the agents,especially through the Philips Curve.

5.1.2 ZLB on Inflation Expectations Compared to Forward Guidance

As visible in figure 6 there are times when inflation expectations are below zero. Specifically, in 7 percent of the time inflation expectations are below zero. Therefore, the implementation of a ZLB on inflation expectations will affect the results of the simulation. The ZLB on inflation expectations is defined as:

$$E_t \hat{\pi}_{t+1} = max\left(0, \pi_t(+1)\right) \tag{20}$$

with $\pi_{t(+1)}$ being the inflation rate at $t+1$. To be able to compare the Smets and Wouters (2007) model to the simulations in Section 3, I introduce a central bank that engages in threshold-based forward guidance by keeping interest rates lower for longer when the nominal interest rate is bound to zero. The forward guidance strategy of the central bank is equivalent to section 4. When the nominal interest rate is above zero the central bank influences the economy by setting the nominal interest rate. When the nominal interest rate is below zero and inflation expectations are also below zero the central bank additionally engages in threshold-based forward guidance to bring the inflation rate back up.

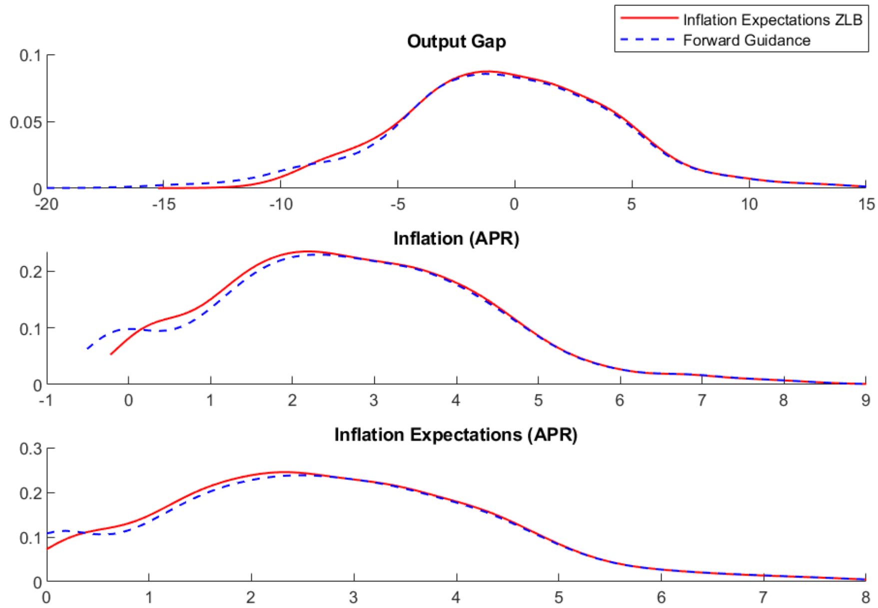

Figure 7: Forward Guidance vs. ZLB on Inflation Expectations
Density function showing the results of a ZLB on inflation expectations vs. an economy with a central bank that engages in forward guidance in the Smets and Wouters model.

Figure 7 shows that in the Smets and Wouters (2007) model including a zero lower bound on inflation expectations or taking into account forward guidance of the central bank leads to very similar simulation results. In both cases, inflation expectations are bound to zero and the realized inflation rate still has a positive probability of being below zero. This matches the findings of the main analysis using the Harding et al. (2022) model.

6 Discussion

Taking all the results from the model simulations I show that the observations of Gorodnichenko and Sergeyev (2021) on the ZLB on inflation expectations can be explained by unconventional monetary policy, in this case by a threshold-based forward guidance strategy. Especially for the distribution of inflation expectations the ZLB on inflation expectations and the forward guidance strategy lead observationally equivalent results, matching with the findings from Gorodnichenko and Sergeyev (2021). This paper is a first step in exploring how their findings can be explained. This becomes especially important when exploring how this observation changes the effects we expect from different types of shocks, but also the efficiency of monetary and fiscal policy measures.

Coming from a theoretical point of view even if there is evidence for a floor of inflation expectations in the data there is no reason to assume that this is due to any underlying fundamentals. The idea of implementing a ZLB on inflation expectations mainly based on empirical observations is in contradiction to the idea already postulated by Lucas (1976). If we now observe a ZLB on inflation expectations and we implement this in a model we can only use it to describe the situation not to forecast future developments. Following Lucas' a central bank that would try to exploit this feature would be bound to find out that this relationship does not exist anymore. This is complemented by the recent findings of Coibion et al. (2021) that consumers realized inflation expectations are sensitive to information.

From a modeling perspective, there is new evidence suggesting that linear models do not perform well predicting negative drops in inflation. Lindé and Trabandt (2018) show that the use of nonlinear macroeconomic models is crucial when analysing large shocks to the economy. They propose to use a non-linear model as a solution to the so-called missing deflation puzzle which states that deflation is observed a lot less often than predicted by the linearized macro models. The use of linearized models is for various reasons still widespread. In this paper the focus is on the observed difference between the realized inflation and the expected inflation rate. The main point of Gorodnichenko and Sergeyev (2021) is that even though inflation rates were negative, inflation expectations still have a floor at zero. This discrepancy in the data is not affected by the macro model definition. For future analysis it will be interesting to consider how the empirical findings of Gorodnichenko and Sergeyev (2021) can be explained in a non-linear model, where negative inflation rates and following negative inflation expectations are less frequent.

This paper shows that when using a linear model, a mechanical floor to inflation expectations is not necessary to explain consumers expectations, rather taking into account central bank policies is very important to explain the differences between consumers inflation expectations and realized inflation rate.

7 Conclusion

In this paper I analyze how a ZLB on inflation expectations impacts the outcomes of an economy over a long time horizon. I find that the ZLB on inflation expectations impacts only the left side of the distribution and shifts the distribution of inflation expectations, inflation and output closer to the steady state. Additionally, I test whether these findings can be rationalized by a central bank conducting forward guidance. In the baseline model, I find the simulation results of an economy with a ZLB on inflation expectations compared to a central bank with a threshold-based forward guidance strategy to be observationally equivalent.

This is confirmed in the simulations using the Smets and Wouters model where introducing a ZLB on inflation expectations and a central bank engaging in forward guidance produce very similar results as well.

For future research it can be interesting to analyze the impact of the shock type on this equivalence result as this paper focuses on demand shocks. Additionally, it may be interesting to examine the implications of a ZLB on inflation expectations in a non-linear model set-up, as the non-linearity already changes the frequency of negative inflation rates.

All told, the empirical findings of Gorodnichenko and Sergeyev (2021) can be explained in a standard New Keynesian macro model, taking into account forward guidance as an unconventional monetary policy tool of the central bank.

8 References

Stéphane Adjemian, Houtan Bastani, Michel Juillard, Fréderic Karamé, Ferhat Mihoubi, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot. Dynare: Reference manual version 5. Dynare Working Papers 72, CEPREMAP, 2022.

Guillermo A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383—-398, 1983.

Larry Christiano, Martin Eichenbaum, and Mathias Trabandt. Understanding the great recession. *American Economic Journal: Macroeconomics*, 7(1):110–167, 2015. doi: dx.doi.org/10.1257/mac.20140104.

Olivier Coibion, Yuriy Gorodnichenko, and Rupal Kamdar. The formation of expectations, inflation, and the phillips curve. *Journal of Economic Literature*, 56 (4):1447–91, 2018.

Olivier Coibion, Yuriy Gorodnichenko, and Michael Weber. Monetary policy communications and their effects on households. *NBER working paper series*, 2021. URL `http://www.nber.org/papers/w25482`.

Yuriy Gorodnichenko and Dmitriy Sergeyev. Zero lower bound on inflation expectations. *NBER working paper*, 2021. doi: 10.3386/w29496. URL `https://www.nber.org/papers/w29496`.

Martin Harding, Jesper Lindé, and Mathias Trabandt. Resolving the missing deflation puzzle. *Journal of Monetary Economics*, 126:15–34, 2022. ISSN 0304-3932. doi: 10.1016/j.jmoneco.2021.09.003.

Lars Jonung. Perceived and expected rates of inflation in swede. *The American economic review*, 71(5):961–968, 1981. ISSN 0002-8282.

Michel Juillard. Dynare : a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm. *IDEAS Working Paper Series from RePEc*, 1996.

M.S. Kimball. The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit, and Banking*, 27:1241–1277, 1995.

Jesper Lindé and Mathias Trabandt. Should we use linearized models to calculate fiscal multipliers? *Journal of Applied Econometrics*, 33(7):937–965, 2018. ISSN 0883-7252. doi: 10.1002/jae.2641. URL `https://onlinelibrary.wiley.com/doi/epdf/10.1002/jae.2641`.

Robert E Lucas. Econometric policy evaluation: A critique. *Carnegie-Rochester conference series on public policy*, 1:19–46, 1976. ISSN 0167-2231. doi: 10.1016/S0167-2231(76)80003-6.

Johannes Pfeiffer. Smets˙wouters˙2007˙45.mod, 2020. URL https://github.com/JohannesPfeifer/DSGE_mod/blob/master/Smets_Wouters_2007/Smets_Wouters_2007_45.mod.

Frank Smets and Rafael Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *The American economic review*, 97(3):586–606, 2007. ISSN 0002-8282. doi: 10.1257/aer.97.3.586.

Lars E.O. Svensson. Forward guidance. *NBER working paper*, 2014. doi: 10.3386/w20796. URL https://www.nber.org/papers/w20796.

A Appendix

A.1 Forward Guidance with a different threshold

As an additional robustness check I change the threshold at which the central bank reacts to the nominal interest rate hitting the zero lower bound and the expected inflation rate dropping as well. In the main simulation the central bank reacts with forward guidance if the expected inflation rate drops below zero. Now the central bank will only react if the expected inflation rate falls below -0.5 percent.
This means that even in a scenario where a central bank would be more cautious using forward guidance as a policy instrument it can still bring inflation expectations up. Figure 8 shows that the results are overall robust to this change and may even better explain the realization of negative interest rates.

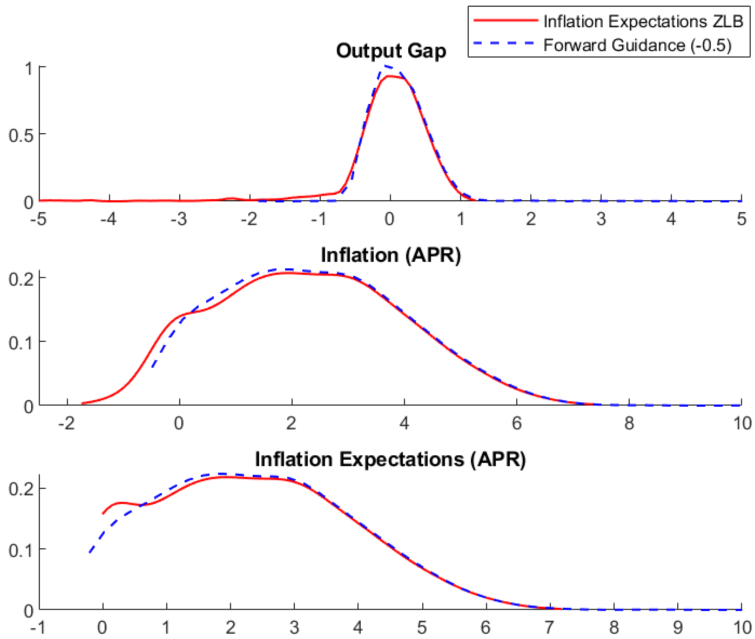

Figure 8: Forward Guidance vs. ZLB on Inflation Expecations
Density plot showing output, inflation and inflation expectations of a ZLB on inflation expectations compared to an economy with a central bank that reacts with forward guidance to inflation expectation rates below -0.5 percent.

A.2 Forward Guidance Strategy

Figure 9 gives a short overview of the central banks forward guidance strategy.

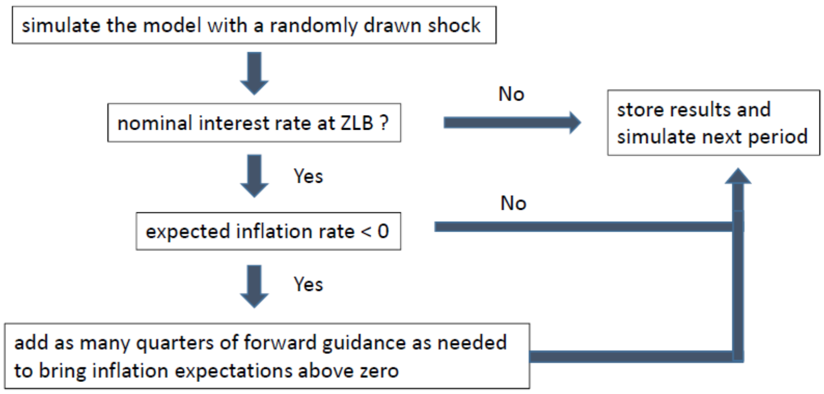

Figure 9: Forward Guidance Decision of the Central Bank

A.3 The Baseline Model

A.3.1 Log-Linearized Set of Equations

More details can be found in the technical appendix of Lindé and Trabandt (2018).

$$\text{Labor/leisure:} \quad \chi\,\hat{n}_t + \frac{\hat{c}_t - \breve{\nu}_t}{\sigma\,(1-\nu)} = \hat{w}_t - \frac{\breve{\tau}_{N\,t}}{1-\tau_N} \tag{A.1}$$

$$\text{Euler equation:} \quad 0 = \hat{R}_t - E_t\hat{\Pi}_{t+1} - \frac{\hat{c}_t - \breve{\nu}_t}{\sigma\,(1-\nu)} + - \frac{E_t\hat{c}_{t+1} - E_t\breve{\nu}_{t+1}}{\sigma\,(1-\nu)} \tag{A.2}$$

$$\text{Resource constraint:} \quad \hat{c}_t\,c + \hat{g}_t\,g = \hat{y}_t\,y \tag{A.3}$$

$$\text{Production:} \quad \hat{y}_t = \hat{n}_t\,(1-\alpha) \tag{A.4}$$

$$\text{Phillips curve:} \quad \hat{\Pi}_t = E_t\hat{\Pi}_{t+1}\,\beta + \frac{\frac{(1-\xi_p)\,(1-\beta\,\xi_p)}{\xi_p}}{1-(1+\theta_p)\,\psi}\,\hat{mc}_t \tag{A.5}$$

$$\text{Marginal cost:} \quad \hat{mc}_t = \hat{w}_t + \hat{n}_t\,\alpha \tag{A.6}$$

$$\text{Taylor rule:} \quad \hat{R}_t = max(\tfrac{1-R}{R}, \hat{\pi}_t\,\gamma_\pi + \gamma_x\,\left(\hat{y}_t - \hat{y}_t^{pot}\right)) \tag{A.7}$$

$$\text{Government budget:} \quad \begin{aligned} \breve{b}_t = {} & g\,\hat{g}_t + \tfrac{R}{\Pi}\,g\,\left(\hat{R}_{t-1} - \hat{\pi}_t\right) + \tfrac{R}{\Pi}\,\breve{b}_{t-1} - \hat{c}_t\,c\,\tau_C \\ & - \breve{\tau}_{N,t}\,w\,n - n\,\tau_N\,w\,(\hat{n}_t + \hat{w}_t) - \breve{\tau}_t \end{aligned} \tag{A.8}$$

$$\text{Fiscal rule 1:} \quad \breve{\tau}_t = \breve{b}_{t-1}\,\varphi_T \tag{A.9}$$

$$\text{Fiscal rule 2:} \quad \breve{\tau}_{N\,t} = \breve{b}_{t-1}\,\varphi_N \tag{A.10}$$

$$\text{Flex-price euler equation:} \quad 0 = \hat{rr}_t^{pot} - \frac{E_t\hat{c}_{t+1}^{pot} - E_t\breve{\nu}_{t+1}}{\sigma\,(1-\nu)} + \frac{\hat{c}_t^{pot} - \breve{\nu}_t}{\sigma\,(1-\nu)} \tag{A.11}$$

$$\text{Flex-price labor/leisure:} \quad \frac{\hat{c}_t^{pot} - \breve{\nu}_t}{\sigma\,(1-\nu)} + \chi\,\hat{n}_t^{pot} = \hat{w}_t^{pot} - \frac{\breve{\tau}_{N,t}^{pot}}{1-\tau_N} \tag{A.12}$$

$$\text{Flex-price real wage:} \quad \hat{w}_t^{pot} = -\alpha\,\hat{n}_t^{pot} \tag{A.13}$$

$$\text{Flex-price resource constraint:} \quad g\,\hat{g}_t + c\,\hat{c}_t^{pot} = y\,\hat{y}_t^{pot} \tag{A.14}$$

$$\text{Flex-price production:} \quad \hat{y}_t^{pot} = (1-\alpha)\,\hat{n}_t^{pot} \tag{A.15}$$

$$\text{Flex-price gov. budget:} \quad \begin{aligned} \breve{b}_t^{pot} = {} & g\,\hat{g}_t + \tfrac{R}{\Pi}\,B\,\hat{rr}_{t-1}^{pot} + \tfrac{R}{\Pi}\,\breve{b}_{t-1}^{pot} - c\,\tau_C\,\hat{c}_t^{pot} \\ & - w\,n\,\breve{\tau}_{N\ t}^{pot} - n\,\tau_N\,w\,\left(\hat{n}_t^{pot} + \hat{w}_t^{pot}\right) - \breve{\tau}_t^{pot} \end{aligned} \tag{A.16}$$

$$\text{Flex-price fiscal rule 1:} \quad \breve{\tau}_t^{pot} = \phi_T\,\breve{b}_{t-1}^{pot} \tag{A.17}$$

$$\text{Flex-price fiscal rule 2:} \quad \breve{\tau}_{N\ t}^{pot} = \phi_N\,\breve{b}_{t-1}^{pot} \tag{A.18}$$

$$\text{Output Gap:} \quad \breve{x}_t = \hat{y}_t - \hat{y}_t^{pot} \tag{A.19}$$

$$\text{Real Interest Rate:} \quad \hat{rr}_t = \hat{R}_t - E_t\hat{\Pi}_{t+1} \tag{A.20}$$

$$\text{Consumption Demand Shock:} \quad \breve{\nu}_t = \rho_\nu\,\breve{\nu}_{t-1} + \varepsilon_{\nu t} \tag{A.21}$$

A.3.2 Parameters

Table 2: Parameter Values

Parameter	Value	Description
$\bar{\pi}$	1.005	steady state gross inflation rate
θ_p	0.1	net price markup in steady state
ξ_p	0.667	Calvo sticky prices parameter
ψ	-12.2	Kimball aggregator parameter
γ_π	1.5	Taylor rule coefficient on inflation
γ_x	0.125	Taylor rule coefficient on output gap
α	0.3	capital share in production
β	0.995	discount factor of households
σ	1	inverse intertemporal elasticity of substitution
ψ	2.5	inverse Frisch elasticity of labor supply
ν	0.01	steady state of cons. demand shock
τ_C	0.0	steady state consumption tax
τ_y	0.0	steady state transfer to GDP ratio
ϕ_T	0.01	fiscal rule for lump-sum taxes: coefficient on debt
ρ_G	0.95	AR(1) gov. cons shock
ρ_ν	0.95	AR(1) cons. demand shock

A.4 The Smets and Wouters (2007) model

For more detailed derivations please use Smets and Wouters (2007) and Pfeiffer (2020).

A.4.1 Linearized Set of Equations

Flex-price: FOC labor

$$\varepsilon_{at} = \alpha\, {r^{k,flex}}_t + (1-\alpha)\ {w^{flex}}_t \tag{A.22}$$

Flex-price: FOC capital utilization

$${z^{flex}}_t = {r^{k,flex}}_t\, \frac{1}{\frac{\psi}{1-\psi}} \tag{A.23}$$

Flex-price: FOC capital

$${r^{k,flex}}_t = {w^{flex}}_t + {l^{flex}}_t - {k^{s,flex}}_t \tag{A.24}$$

Flex-price: capital services

$$k^{s,flex}{}_t = z^{flex}{}_t + k^{flex}{}_{t-1} \tag{A.25}$$

Flex-price: investment euler equation

$$i^{flex}{}_t = \frac{1}{1 + c_t^{\gamma}\, \bar{c}_t^{\beta}} \left(i^{flex}{}_{t-1} + c_t^{\gamma}\, \bar{c}_t^{\beta}\, i^{flex}{}_{t+1} + \frac{1}{c_t^{\gamma 2}\, \varphi}\, q^{flex}{}_t \right) + \varepsilon^i{}_t \tag{A.26}$$

Flex-price: arbitrage equation capital

$$q^{flex}{}_t = \left(-r^{flex}{}_t\right) + c_2 * \varepsilon^b_{tt}\, \frac{1}{\frac{1-\frac{\lambda}{c_t^{\gamma}}}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)}} + \frac{c_t^{rk}}{1-\delta+c_t^{rk}}\, r^{k,flex}{}_{t+1} + \frac{1-\delta}{1-\delta+c_t^{rk}}\, q^{flex}{}_{t+1} \tag{A.27}$$

Flex-price: consumption euler equation

$$\begin{aligned} c^{flex}{}_t = c_2 * \varepsilon^b_{tt} + \frac{\frac{\lambda}{c_t^{\gamma}}}{1+\frac{\lambda}{c_t^{\gamma}}}\, c^{flex}{}_{t-1} + \frac{1}{1+\frac{\lambda}{c_t^{\gamma}}}\, c^{flex}{}_{t+1} \\ + \frac{(\sigma_c - 1)\, c_t^{whlc}}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)} \left(l^{flex}{}_t - l^{flex}{}_{t+1}\right) - r^{flex}{}_t\, \frac{1-\frac{\lambda}{c_t^{\gamma}}}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)} \end{aligned} \tag{A.28}$$

Flex-price: resource constraint

$$y^{flex}{}_t = c_t^{cy}\, c^{flex}{}_t + i^{flex}{}_t\, c_t^{iy} + \varepsilon^g{}_t + z^{flex}{}_t\, c_t^{rkky} \tag{A.29}$$

Flex-price: production

$$y^{flex}{}_t = \phi_p \left(\varepsilon_{at} + \alpha\, k^{s,flex}{}_t + (1-\alpha)\; l^{flex}{}_t\right) \tag{A.30}$$

Flex-price: wage

$$w^{flex}{}_t = l^{flex}{}_t\, \sigma_l + c^{flex}{}_t\, \frac{1}{1-\frac{\lambda}{c_t^{\gamma}}} - c^{flex}{}_{t-1}\, \frac{\frac{\lambda}{c_t^{\gamma}}}{1-\frac{\lambda}{c_t^{\gamma}}} \tag{A.31}$$

Flex-price: law of motion capital

$$k^{flex}{}_t = k^{flex}{}_{t-1} \left(1 - c_t^{ikbar}\right) + i^{flex}{}_t\, c_t^{ikbar} + \varepsilon^i{}_t\, c_t^{\gamma 2}\, \varphi\, c_t^{ikbar} \tag{A.32}$$

Sticky-price: FOC labor

$$\mu_{p_t} = \alpha\, r^k{}_t + (1-\alpha)\; w_t - \varepsilon_{at} \tag{A.33}$$

Sticky-price: FOC capacity utilization

$$z_t = \frac{1}{\frac{\psi}{1-\psi}}\, r^k{}_t \tag{A.34}$$

Sticky-price: FOC capital

$$r^k{}_t = w_t + l_t - k^s{}_t \tag{A.35}$$

Sticky-price: capital services

$$k^s{}_t = z_t + k_{t-1} \tag{A.36}$$

Sticky-price: investment euler equation

$$i_t = \varepsilon^i{}_t + \frac{1}{1 + c_t^{\gamma}\,\bar{c}_t^{\beta}} \left(i_{t-1} + c_t^{\gamma}\,\bar{c}_t^{\beta}\, i_{t+1} + \frac{1}{c_t^{\gamma 2}\,\varphi}\, q_t \right) \tag{A.37}$$

Sticky-price: capital arbitrage

$$q_t = c_2 * \varepsilon^b_{t\,t} \frac{1}{\frac{1-\frac{\lambda}{c_t^{\gamma}}}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)}} + (-r_t) + \pi_{t+1} + \frac{c_t^{rk}}{1-\delta+c_t^{rk}}\, r^k{}_{t+1} + \frac{1-\delta}{1-\delta+c_t^{rk}}\, q_{t+1} \tag{A.38}$$

Sticky-price: consumption euler equation

$$\begin{aligned} c_t = c_2 * \varepsilon^b_{t\,t} + \frac{\frac{\lambda}{c_t^{\gamma}}}{1+\frac{\lambda}{c_t^{\gamma}}}\, c_{t-1} + \frac{1}{1+\frac{\lambda}{c_t^{\gamma}}}\, c_{t+1} \\ + \frac{(\sigma_c - 1)\; cwhlc_t}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)} \left(l_t - l_{t+1}\right) - \frac{1-\frac{\lambda}{c_t^{\gamma}}}{\sigma_c \left(1+\frac{\lambda}{c_t^{\gamma}}\right)} \left(r_t - \pi_{t+1}\right) \end{aligned} \tag{A.39}$$

Sticky-price: rescource constraint

$$y_t = \varepsilon^g{}_t + c_t^{cy}\, c_t + c_t^{iy}\, i_t + c_t^{rkky}\, z_t \tag{A.40}$$

Sticky-price: production function

$$y_t = \phi_p \left(\varepsilon_{at} + \alpha\, k^s{}_t + (1-\alpha)\; l_t\right) \tag{A.41}$$

New Keynesian Phillips curve

$$\pi_t = \frac{1}{1 + c_t^{\gamma}\,\bar{c}_t^{\beta}\,\iota_p} \left(c_t^{\gamma}\,\bar{c}_t^{\beta}\,\pi_{t+1} + \iota_p\,\pi_{t-1} + \mu_{p\,t} \frac{\frac{(1-\xi_p)\left(1-c_t^{\gamma}\,\bar{c}_t^{\beta}\,\xi_p\right)}{\xi_p}}{1+(\phi_p - 1)\;\varepsilon_p} \right) + \varepsilon^p{}_t \tag{A.42}$$

Wage Phillips curve

$$\begin{aligned} w_t = \frac{1}{1+c_t^{\gamma}\,\bar{c}_t^{\beta}}\, w_{t-1} + \frac{c_t^{\gamma}\,\bar{c}_t^{\beta}}{1+c_t^{\gamma}\,\bar{c}_t^{\beta}}\, w_{t+1} + \pi_{t-1} \frac{\iota_w}{1+c_t^{\gamma}\,\bar{c}_t^{\beta}} - \pi_t \frac{1+c_t^{\gamma}\,\bar{c}_t^{\beta}\,\iota_w}{1+c_t^{\gamma}\,\bar{c}_t^{\beta}} \\ + \pi_{t+1} \frac{c_t^{\gamma}\,\bar{c}_t^{\beta}}{1+c_t^{\gamma}\,\bar{c}_t^{\beta}} + \frac{(1-\xi_w)\left(1-c_t^{\gamma}\,\bar{c}_t^{\beta}\,\xi_w\right)}{\left(1+c_t^{\gamma}\,\bar{c}_t^{\beta}\right)\,\xi_w} \frac{1}{1+(\phi_w - 1)\;\varepsilon_w} \left(\sigma_l\, l_t + \frac{1}{1-\frac{\lambda}{c_t^{\gamma}}}\, c_t \right. \\ \left. - \frac{\frac{\lambda}{c_t^{\gamma}}}{1-\frac{\lambda}{c_t^{\gamma}}}\, c_{t-1} - w_t \right) + \varepsilon^w{}_t \end{aligned} \tag{A.43}$$

Taylor rule

$$
\begin{aligned}
{r^*}_t = {} & \pi_t \, r_\pi \, (1-\rho) + (1-\rho) \, r_y \, \left(y_t - {y^{flex}}_t\right) \\
& + r_{\Delta y} \, \left(y_t - {y^{flex}}_t - y_{t-1} + {y^{flex}}_{t-1}\right) + \rho \, {r^*}_{t-1} + {\varepsilon^r}_t
\end{aligned} \tag{A.44}
$$

$$
r_t = max(\frac{1 - r^*}{r^*}, {r^*}_t) \tag{A.45}
$$

Law of motion for productivity

$$
\varepsilon_{at} = \rho_a \, \varepsilon_{at-1} + {\eta^a}_t \tag{A.46}
$$

Law of motion for risk premium

$$
c_2 * \varepsilon^b_{tt} = \rho_b \, c_2 * \varepsilon^b_{tt-1} + {\eta^b}_t \tag{A.47}
$$

Law of motion for spending process

$$
{\varepsilon^g}_t = \rho_g \, {\varepsilon^g}_{t-1} + {\eta^g}_t + {\eta^a}_t \, \rho_{ga} \tag{A.48}
$$

Law of motion for investment specific technology shock process

$$
{\varepsilon^i}_t = \rho_i \, {\varepsilon^i}_{t-1} + {\eta^i}_t \tag{A.49}
$$

Law of motion for monetary policy shock process

$$
{\varepsilon^r}_t = \rho_r \, {\varepsilon^r}_{t-1} + {\eta^m}_t \tag{A.50}
$$

Law of motion for price markup shock process

$$
{\varepsilon^p}_t = \rho_p \, {\varepsilon^p}_{t-1} + {\eta^{p,aux}}_t - \mu_p \, {\eta^{p,aux}}_{t-1} \tag{A.51}
$$

$$
{\eta^{p,aux}}_t = {\eta^p}_t \tag{A.52}
$$

Law of motion for wage markup shock process

$$
{\varepsilon^w}_t = \rho_w \, {\varepsilon^w}_{t-1} + {\eta^{w,aux}}_t - \mu_w \, {\eta^{w,aux}}_{t-1} \tag{A.53}
$$

$$
{\eta^{w,aux}}_t = {\eta^w}_t \tag{A.54}
$$

Law of motion for capital

$$
k_t = \left(1 - c_t^{ikbar}\right) \, k_{t-1} + c_t^{ikbar} \, i_t + {\varepsilon^i}_t \, \varphi \, c_t^{\gamma 2} \, c_t^{ikbar} \tag{A.55}
$$